

A Flavor Kit for BSM models

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Abstract We present a new kit for the study of flavor observables in models beyond the standard model. The setup is based on the public codes **SARAH** and **SPheno** and allows for an easy implementation of new observables. The Wilson coefficients of the corresponding operators in the effective lagrangian are computed by **SPheno** modules written by **SARAH**. New operators can also be added by the user in a modular way. For this purpose a handy **Mathematica** package called **PreSARAH** has been developed. This uses **FeynArts** and **FormCalc** to derive the generic form factors at tree- and 1-loop levels and to generate the necessary input files for **SARAH**. This framework has been used to implement $\text{BR}(\ell_\alpha \rightarrow \ell_\beta \gamma)$, $\text{BR}(\ell_\alpha \rightarrow 3 \ell_\beta)$, $\text{CR}(\mu - e, A)$, $\text{BR}(\tau \rightarrow P \ell)$, $\text{BR}(h \rightarrow \ell_\alpha \ell_\beta)$, $\text{BR}(Z \rightarrow \ell_\alpha \ell_\beta)$, $\text{BR}(B_{s,d}^0 \rightarrow \ell \bar{\ell})$, $\text{BR}(\bar{B} \rightarrow X_s \gamma)$, $\text{BR}(\bar{B} \rightarrow X_s \ell \bar{\ell})$, $\text{BR}(\bar{B} \rightarrow X_{d,s} \nu \bar{\nu})$, $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $\Delta M_{B_s, B_d}$, ΔM_K , ε_K , $\text{BR}(B \rightarrow K \mu \bar{\mu})$, $\text{BR}(B \rightarrow \ell \nu)$, $\text{BR}(D_s \rightarrow \ell \nu)$ and $\text{BR}(K \rightarrow \ell \nu)$ in **SARAH**. Predictions for these observables can now be obtained in a wide range of SUSY and non-SUSY models. Finally, the user can use the same approach to easily compute additional observables.

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1 Introduction

With the exploration of the terascale, particle physics has entered a new era. On the one hand, the discovery of a Higgs boson at the LHC [1, 2] seemingly completed the Standard Model (SM) of particle physics, even though there is still quite some room for deviations from the SM predictions. The observed mass of about 125 GeV in combination with a top quark mass of 173.34 GeV [3] implies within the SM that we potentially live in a metastable vacuum [4]. This, together with other observations, like the dark matter relic density or the unification of gauge forces, indicates that there is physics beyond the SM (BSM). Although no sign of new physics has been found so far at the LHC, colliders are not the only places where one can search for new physics. Low energy experiments focused on flavor observables can also play a major role in this regard, since new particles leave their traces via quantum effects in flavor violating processes such as $b \rightarrow s\gamma$, $B_s \rightarrow \mu^+\mu^-$ or $\mu \rightarrow e\gamma$. In the last few years there has been a tremendous progress in this field, both on the experimental as well as on the theoretical side. In particular, observables from the Kaon- and B-meson sectors, rare lepton decays and electric dipole moments have put stringent bounds on new flavor mixing parameters and/or additional phases in models beyond the SM.

There are several public tools on the market which predict the rates of several flavor observables: **superiso** [5–7], **SUSY_Flavor** [8, 9], **NMSSM-Tools** [10], **MicrOmegas** [11–15], **SuperBSG** [16], **SupeLFV** [17], **SuseFlav** [18], **IsaJet** with **IsaTools** [19–24] or **SPheno** [25, 26]. However, all of these codes have in common that they are only valid in the Two-Higgs-doublet model or in the MSSM or simple extensions of it (NMSSM, bilinear R-parity violation). In addition, none of these tools can be easily extended by the user to calculate additional observables. This has made flavor studies beyond the SM a cumbersome task. The situation has changed with the development of **SARAH** [27–31]. This **Mathematica** package can be used to generate modules for **SPheno**, which then can calculate flavor observables at the 1-loop level in a wide range of supersymmetric and non-supersymmetric models [32–34]. However, so far all the information about the underlying Wilson coefficients¹ for the operators triggering the flavor violation as well as the calculation of the flavor observables had been hardcoded in **SARAH**. Therefore, it was also very difficult for the user to extend the list of calculated observables. The implementation of new operators was even more difficult.

We present a new kit for the study of flavor observables beyond the standard model. In contrast to previous flavor codes, **FlavorKit** is not restricted to a single model, but can be used to obtain predictions for flavor observables in a wide range of models (SUSY and non-SUSY). **FlavorKit** can be used in two different ways. The basic usage of **FlavorKit** allows for the computation of a large number of lepton and quark flavor observables, using generic analytical expressions for the Wilson coefficients of the relevant operators. The setup is based on the public codes **SARAH** and **SPheno**, and thus allows for the analytical and numerical computation of the observables in the model defined by the user. If necessary, the user can also go beyond the basic usage and define his own operators and/or observables. For this purpose, a **Mathematica** package called **PreSARAH** has been developed. This tool uses **FeynArts/FormCalc** [35–40] to compute generic expressions for the required Wilson coefficients at the tree- and 1-loop levels. Similarly, the user can easily implement new observables. With all these tools properly combined, the user can obtain analytical and numerical results for the observables of his interest in the model of his choice. To calculate new flavor observables with **SPheno** for a given model the user only needs the definition of the operators and the corresponding expressions for the observables as well as the model file for **SARAH**. All necessary calculations are done automatically. We have used this setup to implement $\text{BR}(\ell_\alpha \rightarrow \ell_\beta \gamma)$, $\text{BR}(\ell_\alpha \rightarrow 3 \ell_\beta)$, $\text{CR}(\mu - e, A)$, $\text{BR}(\tau \rightarrow P \ell)$, $\text{BR}(h \rightarrow \ell_\alpha \ell_\beta)$, $\text{BR}(Z \rightarrow \ell_\alpha \ell_\beta)$, $\text{BR}(B_{s,d}^0 \rightarrow \ell \bar{\ell})$, $\text{BR}(\bar{B} \rightarrow X_s \gamma)$, $\text{BR}(\bar{B} \rightarrow X_s \ell \bar{\ell})$, $\text{BR}(\bar{B} \rightarrow X_{d,s} \nu \bar{\nu})$, $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu})$, $\Delta M_{B_s, B_d}$, ΔM_K , ε_K , $\text{BR}(B \rightarrow K \mu \bar{\mu})$, $\text{BR}(B \rightarrow \ell \nu)$, $\text{BR}(D_s \rightarrow \ell \nu)$ and $\text{BR}(K \rightarrow \ell \nu)$ in **SARAH**.

This manual is structured as follows: in the next section we give a brief introduction into the calculation of flavor observables focusing on the main steps that one has to follow. Then we present **FlavorKit**, our setup to combine **FeynArts/FormCalc**, **SPheno** and **SARAH** in Section 3. In Section 4 we explain how new observables can be added and in Section 5 how the list of operators can be extended by the user. A comparison between **FlavorKit** and the other public codes is presented in Section 6 taking the MSSM as an example before we

¹Sometimes the Wilson coefficients are also referred to as form factors. We will nevertheless stick to the name Wilson coefficients in the following, also for lepton flavor violating processes.

conclude in Section 7. The appendix contains information about the existing operators and how they have been combined to compute the different flavor observables.

2 General strategy: calculation of flavor observables in a nutshell

Once we have chosen a BSM model ², our general strategy for the computation of a flavor observable follows these steps:

- **Step 1:** We first consider an effective Lagrangian that includes the operators relevant for the flavor observable of our interest,

$$\mathcal{L}_{eff} = \sum_i C_i \mathcal{O}_i. \quad (1)$$

This Lagrangian consists of a list of (usually) higher-dimensional operators \mathcal{O}_i . The Wilson coefficients C_i can be induced either at tree or at higher loop levels and include both the SM and the BSM contributions ($C_i = C_i^{\text{SM}} + C_i^{\text{BSM}}$). They encode the physics of our model.

- **Step 2:** The Wilson coefficients are computed diagrammatically, taking into account all possible tree-level and 1-loop topologies leading to the \mathcal{O}_i operators ³.
- **Step 3:** The results for the Wilson coefficients are plugged in a general expression for the observable and a final result is obtained.

The user has to make a choice in step 1. The list of operators in the effective Lagrangian can be restricted to the most relevant ones or include additional operators beyond the leading contribution, depending on the required level of precision. Usually, the complete set of renormalizable operators contributing to the observable of interest is considered, although in some well motivated cases one may decide to concentrate on a smaller subset of operators. This freedom is not present in step 2. Once the list of operators has been arranged, the computation of the corresponding C_i coefficients follows from the consideration of all topologies (penguin diagrams, box diagrams, ...) leading to the \mathcal{O}_i operators. This is the most complicated and model dependent step, since it demands a full knowledge of all masses and vertices in the model under study. Furthermore, it may be necessary to compute the coefficients at an energy scale and then obtain, by means of their renormalization group running, their values at a different scale. Finally, step 3 is usually quite straightforward since, like step 1, is model independent. In fact, the literature contains general expressions for most flavor observables, thus facilitating the final step. However, one should be aware that the formulas given in the literature assume that certain operators contribute only sub-dominantly and, thus, omit the corresponding contributions. This is in general justified for the SM but not in a general BSM model. In particular, this is the case for processes involving external neutrinos, which are often assumed to be purely left-handed, making the operators associated to their right-handed components to be neglected.

We will exemplify our strategy using a simple example: $\text{BR}(\mu \rightarrow e\gamma)$ in the Standard Model extended by right-handed neutrinos and Dirac neutrino masses. The starting point is, as explained above, to choose the relevant operators. In this case, it is well known that only dipole interactions can contribute to the radiative decay $\ell_\alpha \rightarrow \ell_\beta \gamma$ at leading order ⁴. Therefore, the relevant operators are contained in the $\ell - \ell - \gamma$ dipole interaction Lagrangian. This is in general given by

$$\mathcal{L}_{\ell\ell\gamma}^{\text{dipole}} = ie m_{\ell_\alpha} \bar{\ell}_\beta \sigma^{\mu\nu} q_\nu \left(K_2^L P_L + K_2^R P_R \right) \ell_\alpha A_\mu + \text{h.c.} \quad (2)$$

Here e is the electric charge, q the photon momentum, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are the usual chirality projectors and $\ell_{\alpha,\beta}$ denote the lepton flavors. This concludes step 1.

The information about the underlying model is encoded in the coefficients $K_2^{L,R}$. In the next step, these coefficients have to be calculated by summing up all Feynman diagrams contributing at a given loop level.

²The current version of **FlavorKit** can only handle renormalizable operators at this stage of the computation.

³In principle, one can go beyond the 1-loop level, although in our case we will restrict our computation to the addition of a few NLO corrections.

⁴At next to leading order, one would also have to consider operators like $\bar{\mu}\gamma_\nu e \bar{q}\gamma^\nu q$, to be combined with a $q - q - \gamma$ dipole interaction.

Expressions for these coefficients for many different models are available in the literature. In the SM only neutrino loops contribute and one finds [41]

$$K_2^L = \frac{G_F}{2\sqrt{2}\pi^2} m_\mu \sum_i \lambda_{i\mu} \lambda_{ie}^* (F_1 + F_2) \quad (3)$$

$$K_2^R = \frac{G_F}{2\sqrt{2}\pi^2} m_e \sum_i \lambda_{i\mu} \lambda_{ie}^* (F_1 - F_2) \quad (4)$$

Here, λ_{ij} denote the entries of the Pontecorvo-Maki-Nakagawa-Sakata matrix and F_1 and F_2 are loop functions. One finds approximately $F_1 \simeq -\frac{1}{4} \left(\frac{m_\nu}{m_W}\right)^2$ and $F_2 \simeq 0$. Finally, we just need to proceed to the last step, the computation of the observable. After computing the Wilson coefficients $K_2^{L,R}$ it is easy to relate them to $\text{BR}(\mu \rightarrow e\gamma)$ by using [42]

$$\Gamma(\ell_\alpha \rightarrow \ell_\beta \gamma) = \frac{\alpha m_{\ell_\alpha}^5}{4} \left(|K_2^L|^2 + |K_2^R|^2 \right), \quad (5)$$

This expression holds for all models. With this final step, the computation concludes.

As we have seen, the main task to get a prediction for $\text{BR}(\mu \rightarrow e\gamma)$ in a new model is to calculate $K_2^{L,R}$. However, this demands the knowledge of all masses and vertices involved. Moreover, in most cases a numerical evaluation of the resulting loop integrals is also welcome. Therefore, even for a simple process like $\mu \rightarrow e\gamma$, a computation from scratch in a new model can be a hard work. In order to solve this practical problem, we are going to present here a fully automatized way to calculate a wide range of flavor observables for several classes of models.

3 Setup

3.1 FlavorKit: usage and goals

As we have seen, the calculation of flavor observables in a specific model is a very demanding task. A detailed knowledge about the model is required, including

1. expressions for all involved masses and vertices
2. optionally, renormalization group equations to get the running parameters at the considered scale
3. expressions to calculate the operators
4. formulae to obtain the observables from the operators

Nearly all codes devoted to flavor physics have those pieces hardcoded, and they are only valid for a few specific models⁵. The only exception is **SPheno**, thanks to its extendability with new modules for additional models. These modules are generated by the **Mathematica** package **SARAH** and provide all necessary information about the calculation of the (loop corrected) mass spectrum, the vertices and the 2-loop RGEs. These expressions, derived from fundamental principles for any (renormalizable) model, contain all the information required for the computation of flavor observables. In fact, **SARAH** also provides **Fortran** code for a set of flavor observables. For this output, generic expressions of the necessary Wilson coefficients have been included. These are matched to the model chosen by the user and related to the observables by the standard formulae available in the literature. However, it was hardly possible for the user to extend the list of observables or operators included in **SARAH** without a profound knowledge of either the corresponding **Mathematica** or **Fortran** code.

We present a new setup to fill this gap in **SARAH**: **FlavorKit**. As discussed in Sec. 2, the critical step in the computation of a flavor observable is the derivation of analytical expressions for the Wilson coefficients of the relevant operators. This step, being model dependent, requires information about the model spectrum and interactions. However, generic expressions can be derived, later to be matched to the specific spectrum and interaction Lagrangian of a given model. For this purpose, we have created a new **Mathematica** package called **PreSARAH**. This package uses the power of **FeynArts** and **FormCalc** to calculate generic 1-loop amplitudes, to extract the coefficients of the demanded operators, to translate them into the syntax needed for **SARAH** and to write the necessary wrapper code. **PreSARAH** works for any 4-fermion or 2-fermion-1-boson operators and will be extended in the future to include other kinds of operators. The current version already contains a long

⁵Recently, **Peng4BSM@LO** [43] was made public. This code derives analytical expressions for vector penguins for a model defined in the corresponding **FeynArts** model file.

Lepton flavor	Quark flavor
$\ell_\alpha \rightarrow \ell_\beta \gamma$	$B_{s,d}^0 \rightarrow \ell^+ \ell^-$
$\ell_\alpha \rightarrow 3 \ell_\beta$	$\bar{B} \rightarrow X_s \gamma$
$\mu - e$ conversion in nuclei	$\bar{B} \rightarrow X_s \ell^+ \ell^-$
$\tau \rightarrow P \ell$	$\bar{B} \rightarrow X_{d,s} \nu \bar{\nu}$
$h \rightarrow \ell_\alpha \ell_\beta$	$B \rightarrow K \ell^+ \ell^-$
$Z \rightarrow \ell_\alpha \ell_\beta$	$K \rightarrow \pi \nu \bar{\nu}$
	$\Delta M_{B_{s,d}}$
	ΔM_K and ε_K
	$P \rightarrow \ell \nu$

Table 1 List of flavor violating processes and observables which have been already implemented in **FlavorKit**. To the left, observables related to lepton flavor, whereas to the right observables associated to quark flavor. See appendices C.1 and C.2 for the definition of the observables and the relevant references for their calculation.

list of fully implemented operators (see Appendix B). The results for the Wilson coefficients obtained with **PreSARAH** are then interpreted by **SARAH**, which adapts the generic expressions to the specific details of the model chosen by the user and uses snippets of **Fortran** code to calculate flavor observables from the resulting Wilson coefficients. As for the operators, there is a long list of observables already implemented (see Appendices C.1 and C.2). Finally, **SARAH** can be used to obtain analytical output in \LaTeX format or to create **Fortran** modules for **SPheno**, thus making possible numerical studies.

FlavorKit can be used in two ways:

- **Basic usage:** This is the approach to be followed by the user who does not need any operator nor observable beyond what is already implemented in **FlavorKit**. In this case, **FlavorKit** reduces to the standard **SARAH** package. The user can use **SARAH** to obtain analytical results for the flavor observables and, if he wants to make numerical studies, to produce **Fortran** modules for **SPheno**. For the list of implemented operators we refer to Appendix B, whereas the list of implemented observables is given in Table 1.
- **Advanced usage:** This is the approach to be followed by the user who needs an operator or an observable not included in **FlavorKit**. In case the user is interested in an operator that is not implemented in **FlavorKit**, he can define his own operators and get analytical results for their coefficients using **PreSARAH**. Then the output can be passed to **SARAH** in order to continue with the basic usage. In case the user is interested in an observable that is not implemented in **FlavorKit**, this can be easily implemented by the addition of a **Fortran** file, with a few lines of code relating the observable to the operators in **FlavorKit** (implemented by default or added by the user). The **Fortran** files just have to be put together with a short steering file into a specific directory located in the main **SARAH** directory. Then one can continue with the basic usage.

The combination of **PreSARAH** together with **SARAH** and **SPheno** allows for a modular and precise calculation of flavor observables in a wide range of particles physics models. We have summarized the setup in Fig. 1: the user provides as input **SARAH** model files for his favorite models or takes one of the models which are already implemented in **SARAH** (see Appendix D for a list of models available in **SARAH**). New observables are implemented by providing the necessary **Fortran** code to **SARAH** while new operators can be either implemented by hand or by using **PreSARAH** which then calls **FeynArts** and **FormCalc** for the calculation of the necessary diagrams. However, most users will not require to implement new operators or observables. In this case, the user can simply use **SARAH** in the standard way and (1) derive analytical results for the Wilson coefficients and observables, and (2) generate **Fortran** modules for **SPheno** in order to run numerical analysis.

3.2 Download and installation

FlavorKit involves several public codes. We proceed to describe how to download and install them.

1. FeynArts/FormCalc

FeynArts and **FormCalc** can be downloaded from

www.feynarts.de/

It is also possible to use the script **FeynInstall**, to be found on the same site, for an automatic installation.

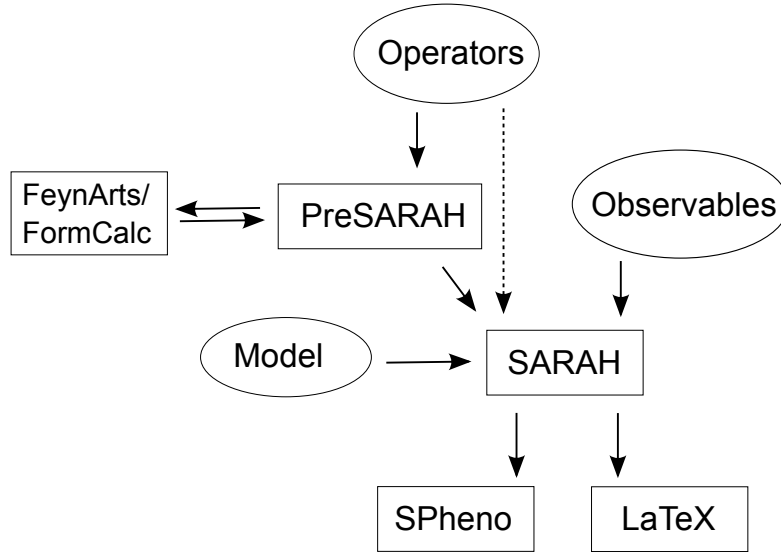


Fig. 1 Schematic way to use **FlavorKit**: the user can define new operators in **PreSARAH**, which then calculates the coefficients in a generic form using **FeynArts** and **FormCalc** and creates the necessary input files for **SARAH**. In addition, **Fortran** code can be provided to relate the Wilson coefficients to specific flavor observables. This information is used by **SARAH** to generate **SPheno** code for the numerical calculation of the observables.

2. SARAH and PreSARAH

SARAH can be downloaded from

sarah.hepforge.org/

No installation or compilation is necessary. Both packages just need to be extracted by using **tar**.

```
> tar -xf SARAH-4.2.0
```

```
> tar -xf PreSARAH-1.0.0
```

PreSARAH needs the paths to load **FeynArts** and **FormCalc**. These have to be provided by the user in the file **PreSARAH.ini**

```
1 FeynArtsPackage = "FeynArts/FeynArts.m";
2 FormCalcPackage = "FormCalc/FormCalc.m";
```

This would work if **FeynArts** and **FormCalc** have been installed in the **Application** directory of the local **Mathematica** installation. Otherwise, absolute paths should be used, e.g.

```
1 FeynArtsPackage = "/home/$user/$path/FeynArts-3.7/FeynArts.m";
2 FormCalcPackage = "/home/$user/$path/FormCalc-8.1/FormCalc.m";
```

3. SPheno

SPheno can be downloaded from

spheno.hepforge.org/

After extracting the package, **make** is used for the compilation.

```
> tar -xf SPheno-3.3.0.tar.gz
```

```
> cd SPheno-3.3.0
```

```
> make
```

3.3 Basic usage

As explained above, **FlavorKit** can be used in several ways, depending on the user's needs and interests. The advanced usage, which involves the introduction of new observables and/or the computation of new operators,

is explained in detail in Secs. 4 and 5. Here we focus on the basic usage, which just requires the codes **SARAH** and **SPheno**.

SARAH can handle the analytical derivation of all the relevant Wilson coefficients in the model defined by the user. The resulting expressions can be then extracted in \LaTeX form or used to generate a **SPheno** module for numerical evaluation. These are the steps to follow in order to use **SARAH**:

1. **Loading SARAH:** after starting **Mathematica**, **SARAH** is loaded via
`<<SARAH-4.2.0/SARAH.m`
or via
`<<[$path]/SARAH-4.2.0/SARAH.m`
The first choice works if **SARAH** has been installed in the **Application** directory of **Mathematica**. Otherwise, the absolute path (`[$path]`) to the local **SARAH** installation must be used.
2. **Initialize a model:** as example for the initialization of a model in **SARAH** we consider the NMSSM:
`Start['NMSSM'];`
3. **Obtaining the \LaTeX output:** the user can get \LaTeX output with all the information about the model (including the coefficients for the flavor operators) via
`ModelOutput[EWSB];`
`MakeTeX[];`
4. **Obtaining the SPheno code:** to create the **SPheno** output the user should run
`MakeSPheno[];`

Thanks to **FlavorKit**, **SARAH** can also write \LaTeX files with the analytical expressions for the Wilson coefficients. These are given individually for each Feynman diagram contributing to the coefficients, and saved in the folder

$$[\$SARAH]/Output/[\$MODEL]/EWSB/TeX/FlavorKit/$$

For the 4-fermion operators the results are divided into separated files for tree-level contributions, penguins contributions and box contributions. The corresponding Feynman diagrams are drawn by using **FeynMF** [44]. To compile all Feynman diagrams at once and to generate the pdf file, a shell script called `MakePDF_[$OPERATOR].sh` is written as well by **SARAH**.

In case the user is interested in the numerical evaluation of the flavor observables, a **SPheno** module must be created as explained above. Once this is done, the resulting **Fortran** code can be used for the numerical analysis of the model. This can be achieved in the following way:

1. **building SPheno:** as soon as the **SPheno** output is finished, open a terminal and enter the root directory of the **SPheno** installation, and create a new subdirectory, copy the **SARAH** output to that directory and compile it
`> cd [$SPheno]`
`> mkdir NMSSM`
`> cp [$SARAH]/Output/NMSSM/EWSB/SPheno/* NMSSM/`
`> make Model=NMSSM`
2. **Running SPheno:** After the compilation, a new binary **SPhenoNMSSM** is created. This file can be executed providing a standard Les Houches input file (**SARAH** provides an example file, see the **SARAH** output folder). Finally, **SPheno** is executed via
`> ./bin/SPhenoNMSSM NMSSM/LesHouches.in.NMSSM`
This generates the output file `SPheno.spc.NMSSM`, which contains the blocks `QFVobservables` and `LFVobservables`. In those two blocks, the results for quark and lepton flavor violating observables are given.

Finally, an even easier way to implement new models in **SARAH** is the **butler** script provided with the **SUSY Toolbox** [45]

sarah.hepforge.org/Toolbox/

3.4 Limitations

FlavorKit is a tool intended to be as general as possible. For this reason, there are some limitations compared to codes which perform specific calculations in a specific model. Here we list the main limitations of **FlavorKit**:

- Chiral resummation is not included because of its large model dependence, see e.g. [46] and references therein.
- Even though we have included some of the higher order corrections for the SM part of some observables in a parametric way, 2- or higher loop corrections, calculated in the context of the SM or the MSSM for specific observables, are not considered, see for instance [47–54].

4 Advanced usage I: Implementation of new observables using existing operators

In order to introduce new observables to the **SPheno** output of **SARAH**, the user can add new definitions to the directories

`[$SARAH]/FlavorKit/[$Type]/Processes/`

`[$Type]` is either **LFV** for lepton flavor violating or **QFV** for quark flavor violating observables. The definition of the new observables consists of two files

1. A steering file with the extension `.m`
2. A **Fortran** body with the extension `.f90`

The steering file contains the following information:

- **NameProcess**: a string as name for the set of observables.
- **NameObservables**: names for the individual observables and numbers which are used to identify them later in the **SPheno** output. The value is a three dimensional list. The first part of each entry has to be a symbol, the second one an integer and the third one a comment to be printed in the **SPheno** output file (`{{name1,number1,comment1},...}`).
- **NeededOperators**: The operators which are needed to calculate the observables. A list with all operators already implemented in **FlavorKit** is given in Appendix B. In case the user needs additional operators, this is explained in Sec. 5.
- **Body**: The name (as string) of the file which contains the **Fortran** code to calculate the observables from the operators.

For instance, the corresponding file to calculate $\ell_\alpha \rightarrow \ell_\beta \gamma$ reads

```

1 NameProcess = "LLpGamma";
2 NameObservables = {{muEgamma, 701, "BR(mu->e gamma)"},
3                     {tauEgamma, 702, "BR(tau->e gamma)"},
4                     {tauMuGamma, 703, "BR(tau->mu gamma)"} };
5 NeededOperators = {K2L, K2R};
6 Body = "LLpGamma.f90";

```

The observables will be saved in the variables `muEgamma`, `tauEgamma`, `tauMuGamma` and will show up in the spectrum file written by **SPheno** in the block **FlavorKitLFV** as numbers 701 to 703.

The file which contains the body to calculate the observables should be standard **Fortran90** code. For our example it reads

```

1 Real(dp) :: width
2 Integer :: i1, gt1, gt2
3
4 Do i1=1,3
5
6 If (i1.eq.1) Then          ! mu -> e gamma
7   gt1 = 2
8   gt2 = 1

```



```

9  Elseif (i1.eq.2) Then      !tau -> e gamma
10     gt1 = 3
11     gt2 = 1
12  Else                      ! tau -> mu gamma
13     gt1 = 3
14     gt2 = 2
15  End if
16
17  width=0.25_dp*mf_l(gt1)**5*(Abs(K2L(gt1,gt2))**2 &
18      & +Abs(K2R(gt1,gt2))**2)*Alpha
19
20  If (i1.eq.1) Then
21     muEgamma = width/(width+GammaMu)
22  Elseif (i1.eq.2) Then
23     tauEgamma = width/(width+GammaTau)
24  Else
25     tauMuGamma = width/(width+GammaTau)
26  End if
27
28  End do

```

`Real(dp)` is the `SPheno` internal definition of double precision variables. Similarly one would have to use `Complex(dp)` for complex double precision variables when necessary.

Besides the operators, the SM parameters given in Table 2 and the hadronic parameters given in Tables 3 and 4 can be used in the calculations. For instance, we used `Alpha` for $\alpha(0)$ and `mf_l` which contains the poles masses of the leptons as well as `GammaMu` and `GammaTau` for the total widths of μ and τ leptons.

Real Variables					
AlphaS_MZ	$\alpha_S(M_Z)$	AlphaS_160	$\alpha_S(Q)$	sinW2	$\sin(\Theta_W)^2$
sinW2_MZ	$\sin(\Theta_W)^2$ at M_Z	sinW2_160	$\sin(\Theta_W)^2$ at Q	Alpha	$\alpha(0)$
Alpha_MZ	$\alpha(M_Z)$	Alpha_160	$\alpha(Q)$	MW	M_W
MW_MZ	$M_W(M_Z)$	MW_160	$M_W(Q)$		
GammaMu	Width Γ_μ of μ	GammaTau	Width Γ_τ of τ		
Real Vectors of length 3					
mf_d_160	$m_d(Q)$	mf_d_MZ	$m_d(M_Z)$	mf_d	m_d
mf_u_160	$m_u(Q)$	mf_u_MZ	$m_u(M_Z)$	mf_u	m_u
mf_l_160	$m_l(Q)$	mf_l_MZ	$m_l(M_Z)$	mf_l	m_l
Complex Arrays of dimension 3×3					
CKM_MZ	CKM at (M_Z)	CKM_160	CKM at Q	CKM	input

Table 2 List of SM parameters available in `FlavorKit`. All hadronic observables are calculated at $Q = 160$ GeV.

By extending or changing the file `hadronic_parameters.m` in the `FlavorKit` directory, it is possible to add new variables for the mass or life time of mesons. These variables are available globally in the resulting `SPheno` code. The numerical values for the hadronic parameters can be changed in the Les Houches input file by using the blocks `FCONST` and `FMASS` defined in the Flavor Les Houches Accord (FLHA) [55].

It may happen that the calculation of a specific observable has to be adjusted for each model. This is for instance the case when (1) the calculation requires the knowledge of the number of generations of fields, (2) the mass or decay width of a particle, calculated by `SPheno`, is needed as input, or (3) a rotation matrix of a specific field enters the analytical expressions for the observable. For these situations, a special syntax has been created. It is possible to start a line with `@` in the `Fortran` file. This line will then be parsed by `SARAH`, and `Mathematica` commands, as well as `SARAH` specific commands, can be used. We made use of this functionality in the implementation of $h \rightarrow \ell_\alpha \ell_\beta$. The lines in `hLLp.f90` read

```
1  ! Check for SM like Higgs
```

Particle	Life time	default [s]	Mass	default [GeV]	PDG number
π^0	tau_pi0	$8.52 \cdot 10^{-17}$	mass_pi0	0.13498	111
π^+	tau_pip	$2.60 \cdot 10^{-8}$	mass_pip	0.13957	211
$\rho(770)^0$	tau_rho0	$4.41 \cdot 10^{-24}$	mass_rho0	0.77549	113
D^0	tau_D0	$4.10 \cdot 10^{-13}$	mass_D0	1.86486	421
D^+	tau_Dp	$1.04 \cdot 10^{-12}$	mass_Dp	1.86926	411
D_s^+	tau_DSsp	$5.00 \cdot 10^{-13}$	mass_DSsp	1.96849	431
D_s^{*+}	tau_DSsp	-	mass_DSsp	2.1123	433
η	tau_eta	$5.06 \cdot 10^{-19}$	mass_eta	0.54785	221
$\eta'(958)$	tau_etap	$3.31 \cdot 10^{-21}$	mass_etap	0.95778	331
$\omega(782)$	tau_omega	$7.75 \cdot 10^{-23}$	mass_omega	0.78265	223
$\phi(1020)$	tau_phi	$1.54 \cdot 10^{-22}$	mass_phi	1.01946	333
K^0	tau_KL0	$5.12 \cdot 10^{-8}$	mass_KL0	-	130
K_S^0	tau_KS0	$0.90 \cdot 10^{-10}$	mass_KS0	-	310
K^0	tau_K0	-	mass_K0	0.49761	311
K^+	tau_Kp	$1.24 \cdot 10^{-8}$	mass_Kp	0.49368	321
B^0	tau_B0d	$1.52 \cdot 10^{-12}$	mass_B0d	5.27958	511
B_d^0	tau_B0s	$1.50 \cdot 10^{-12}$	mass_B0s	5.36677	531
B^+	tau_Bp	$1.64 \cdot 10^{-12}$	mass_Bp	5.27925	521
B^{*0}	tau_B0c	$1.43 \cdot 10^{-23}$	mass_B0c	5.3252	513
B^{*+}	tau_Bpc	$1.43 \cdot 10^{-23}$	mass_Bpc	5.3252	523
B_c^+	tau_Bcp	$4.54 \cdot 10^{-13}$	mass_Bcp	6.277	541
$K^{*0}(892)$	tau_K0c	$1.42 \cdot 10^{-23}$	mass_K0c	0.8959	313
$K^{*+}(892)$	tau_Kpc	$1.30 \cdot 10^{-23}$	mass_Kpc	0.8917	323
$\eta_c(1S)$	tau_etac	$2.22 \cdot 10^{-23}$	mass_etac	2.9810	441
$J/\psi(1S)$	tau_JPsi	$7.08 \cdot 10^{-24}$	mass_JPsi	3096.92	443
$\Upsilon(1S)$	tau_Ups	$1.21 \cdot 10^{-23}$	mass_Ups	9.4603	553

Table 3 Hadronic parameters used in FlavorKit. These can be changed via FMASS and FLIFE in the Les Houches input file.

Decay constant	Variable	default [MeV]	FLHA
f_K	f_k_CONST	176	FCONST[321,1]
f_{K^+}	f_Kp_CONST	156	FCONST[323,1]
f_π	f_pi_CONST	118	FCONST[111,1]
$f_{B_d^0}$	f_B0d_CONST	194	FCONST[511,1]
$f_{B_s^0}$	f_B0s_CONST	234	FCONST[531,1]
f_{B^+}	f_Bp_CONST	234	FCONST[521,1]
$f_{\eta'}$	f_etap_CONST	172	FCONST[231,1]
f_ρ	f_rho_CONST	220	FCONST[213,1]
f_{D^+}	f_Dp_CONST	256	FCONST[411,1]
f_{D_s}	f_Ds_CONST	248	FCONST[431,1]

Table 4 Decay constants available in the SPheno output of SARAH. The values can be changed according to the FLHA conventions using the block FCONST in the Les Houches input file.

```

2 @ If [getGen[HiggsBoson]>1, "hLoc = MaxLoc(Abs(" <> <-
    <- ToString[HiggsMixingMatrix]<>"(2,:),1)", "hLoc = 1"]
3
4 ! Get Higgs mass
5 @ "mh ="<>ToString[SPhenoMass[HiggsBoson]] <> <-
    <- If [getGen[HiggsBoson]>1,"(hLoc)", ""]
6
7 ! Get Higgs width
8 @ "gamh ="<>ToString[SPhenoWidth[HiggsBoson]] <> <-
    <- If [getGen[HiggsBoson]>1,"(hLoc)", ""]

```

In this implementation we define an integer `hLoc` that gives the generation index of the SM-like Higgs, to be found among all CP even scalars. In the first line it is checked if more than one scalar Higgs is present. If this is the case, the `hLoc` is set to the component which has the largest amount of the up-type Higgs, if not, it is just put to 1. Of course, this assumes that the electroweak basis in the Higgs sector is always defined as

<code>getGen[x]</code>	returns the number of generations of a particle <code>x</code>
<code>getDim[x]</code>	returns the dimension of a variable <code>x</code>
<code>SPhenoMass[x]</code>	returns the name used for the mass of a particle <code>x</code> in the <code>SPheno</code> output
<code>SPhenoMassSq[x]</code>	returns the name used for the mass squared of a particle <code>x</code> in the <code>SPheno</code> output
<code>SPhenoWidth[x]</code>	returns the name used for the width of a particle <code>x</code> in the <code>SPheno</code> output
<code>HiggsMixingMatrix</code>	name of the mixing matrix for the CP even Higgs states in a given model
<code>PseudoScalarMixingMatrix</code>	name of the mixing matrix for the CP odd Higgs states in a given model

Table 5 SARAH commands which can be used in the input file for the calculation of an observable.

(ϕ_d, ϕ_u, \dots) as is the case for all models delivered with SARAH. In the second and third lines, the variables `mh` and `gamh` are set to the mass and total width of the SM-like Higgs, respectively. For this purpose, the SARAH commands `SPhenoMass[x]` and `SPhenoWidth[x]` are used. They return the name of the variable for the mass and width in `SPheno` and it is checked if these variables are arrays or not ⁶. For the MSSM, the above lines lead to the following code in the `SPheno` output:

```

1 ! Check for SM like Higgs
2 hLoc = MaxLoc(Abs(ZH(2, :)) , 1)
3
4 ! Get Higgs mass
5 mh =Mhh(hLoc)
6
7 ! Get Higgs width
8 gamh =gThh(hLoc)

```

We give in Table 5 the most important SARAH commands which might be useful in this context.

Many more examples are given in Appendix C.1, where we have added all input files for the calculations of flavor observables delivered with SARAH.

5 Advanced usage II: Implementation of new operators

The user can also implement new operators and obtain analytical expressions for their Wilson coefficients. In this case, he will need to use `PreSARAH` which, with the help of `FeynArts` and `FormCalc`, provides generic expressions for the coefficients, later to be adapted to specific models with SARAH.

5.1 Introduction

New operators can be implemented by extending the content of the folder

`[$SARAH]/FlavorKit/[$Type]/Operators/`

In the current version of `FlavorKit`, 3- and 4-point operators are supported. Each operator is defined by a `.m`-file. These files contain information about the external particles, the kind of considered diagrams (tree-level, self-energies, penguins, boxes) as well as generic expressions for the coefficients. These expressions, derived from the generic Feynman diagrams contributing to the coefficients, are written in the form of a `Mathematica` code, which can be used to generate `Fortran` code.

For the automatization of the underlying calculations we have created an additional `Mathematica` package called `PreSARAH`, which can be used to create the files for all 4-fermion as well as 2-fermion-1-boson operators. This package creates not only the infrastructure to include the operators in the `SPheno` output of SARAH but makes also use of `FeynArts` and `FormCalc` to calculate the amplitudes and to extract the coefficient of the demanded operators. It takes into account all topologies depicted in Figs. 2 to 6.

⁶ The user can define in the `parameters.m` and `particles.m` file for a given model in SARAH the particles which should be taken to be the CP-even or CP-odd Higgs and the parameter that corresponds to their rotation matrices. This is done by using the `Description` statements `Higgs` or `Pseudo-Scalar Higgs` as well as `Scalar-Mixing-Matrix` or `Pseudo-Scalar-Mixing-Matrix`. If the particle or parameter needed to calculate an observable is not present or has not been defined, the observable is skipped in the `SPheno` output.

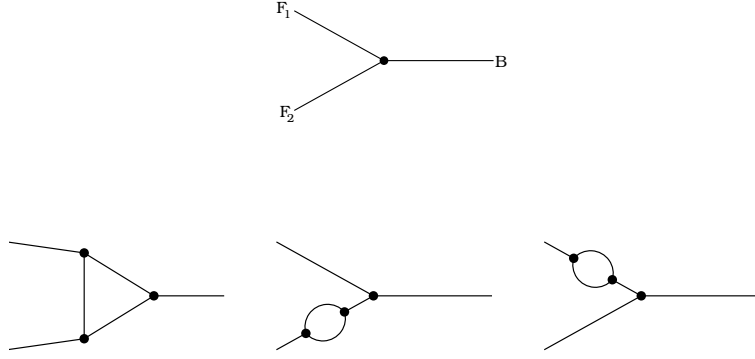


Fig. 2 All topologies considered by PreSARAH to calculate the Wilson coefficients of 2-fermion-1-boson operators. All possible generic combinations of the internal fields are taken into account.

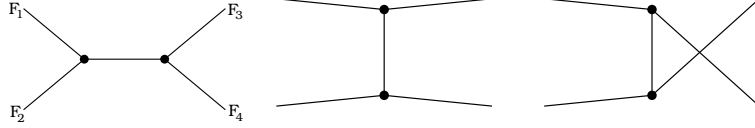


Fig. 3 All tree topologies considered by PreSARAH to calculate the Wilson coefficients of 4-fermion operators. All possible generic combinations of the internal fields are taken into account.

5.2 Input for PreSARAH

In order to derive the results for the Wilson coefficients, PreSARAH needs an input file with the following information:

- **ConsideredProcess**: A string which defines the generic type for the process
 - “4Fermion”
 - “2Fermion1Scalar”
 - “2Fermion1Vector”
- **NameProcess**: A string to uniquely define the process
- **ExternalFields**: The external fields. Possible names are **ChargedLepton**, **Neutrino**, **DownQuark**, **UpQuark**, **ScalarHiggs**, **PseudoScalar**, **Zboson**, **Wboson** ⁷
- **FermionOrderExternal**: the fermion order to apply the Fierz transformation (see the **FormCalc** manual for more details)
- **NeglectMasses**: which external masses can be neglected (a list of integers counting the external fields)
- **ColorFlow**: defines the color flow in the case of four quark operators. To contract the colors of external fields, **ColorDelta** is used, i.e **ColorFlow** = **ColorDelta**[1,2]***ColorDelta**[3,4] assigns $(\bar{q}^\alpha \Gamma q_\alpha)(\bar{q}^\beta \Gamma' q_\beta)$.
- **AllOperators**: a list with the definition of the operators. This is a two dimensional list, where the first entry defines the name of the operator and the second one the Lorentz structure. The operators are expressed in the chiral basis and the syntax for Dirac chains in **FormCalc** is used:

⁷ The **particles.m** file is used to define for each model which particle corresponds to SM states using the **Description** statement together with **Leptons**, **Neutrinos**, **Down-Quarks**, **Up-Quarks**, **Higgs**, **Pseudo-Scalar Higgs**, **Z-Boson**, **W-Boson**. If there is a mixture between the SM particles and other states (like in *R*-parity violating SUSY or in models with additional vector quarks/leptons) the combined state has to be labeled according to the description for the SM state. Notice that in the SM **Pseudo-Scalar Higgs** is just the neutral Goldstone boson. If an external state is not present in a given model or has not been defined as such in the **particles.m** file the corresponding Wilson coefficients are not calculated by **SPheno**.

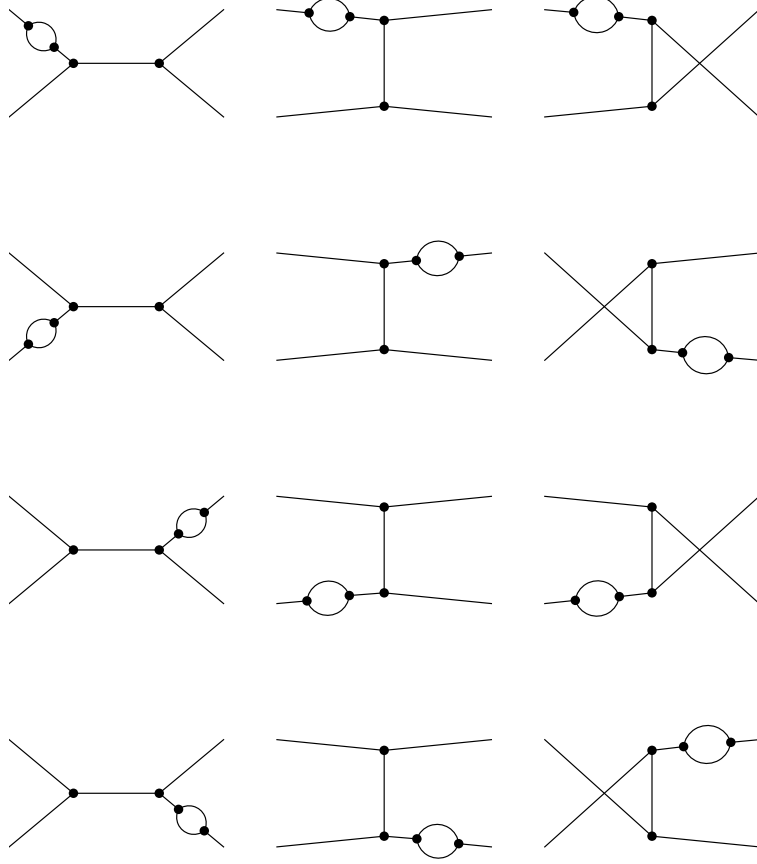


Fig. 4 All self-energy topologies considered by **PreSARAH** to calculate the Wilson coefficients of 4-fermion operators. All possible generic combinations of the internal fields are taken into account.

- 6 for $P_L = \frac{1}{2}(1 - \gamma_5)$, 7 for $P_R = \frac{1}{2}(1 + \gamma_5)$
- **Lor**[1], **Lor**[2] for γ_μ, γ_ν
- **ec**[3] for the helicity of an external gauge boson.
- **k**[N] for the momentum of the external particle N (N is an integer).
- **Pair**[A,B] is used to contract Lorentz indices. For instance, **Pair**[**k**[1],**ec**[3]] stands for $k_\mu^1 \epsilon^{\mu,*}$
- A Dirac chain starting with a negative first entry is taken to be anti-symmetrized.

See the **FormCalc** manual for more details.

To make the definitions more readable, not the full **DiracChain** object of **FeynArts/FormCalc** has to be defined: **PreSARAH** puts everything with the head **Op** into a Dirac chain using the defined fermion order. For 4-fermion operators the combination of both operators is written as dot product. For instance **Op**[6].**Op**[6] is internally translated into

```
DiracChain[Spinor[k[1],MassEx1,-1],6,Spinor[k[2],MassEx2,1]]*
DiracChain[Spinor[k[3],MassEx3,-1],6,Spinor[k[4],MassEx4,1]]
```

while **Op**[6] **Pair**[**ec**[3],**k**[1]] becomes

```
DiracChain[Spinor[k[1],MassEx1,-1],6,Spinor[k[2],MassEx2,1]] Pair[ec[3],k[1]]
```

- **CombinationGenerations**: the combination of external generations for which the operators are calculated by **SPheno**
- **Filters**: a list of filters to drop specific diagrams. Possible entries are **NoBoxes**, **NoPenguins**, **NoTree**, **NoCrossedDiagrams**.

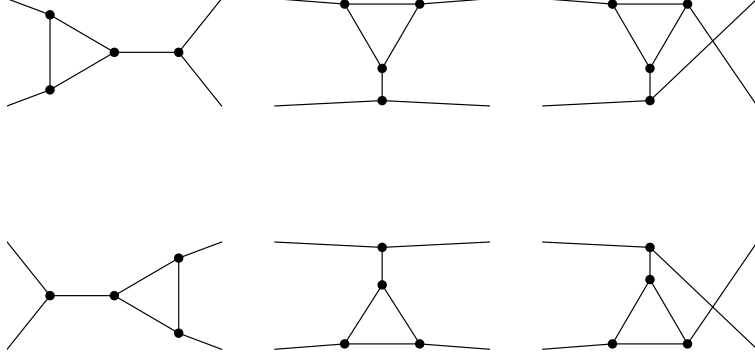


Fig. 5 All penguin topologies considered by **PreSARAH** to calculate the Wilson coefficients of 4-fermion operators. All possible generic combinations of the internal fields are taken into account.

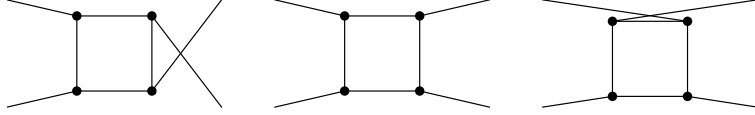


Fig. 6 All box topologies considered by **PreSARAH** to calculate the Wilson coefficients of 4-fermion operators. All possible generic combinations of the internal fields are taken into account.

- **Filters** = {NoBoxes, NoPenguins} can be used for processes which are already triggered at tree-level
- **Filters** = {NoPenguins} might be useful for processes which at the 1-loop level are only induced by box diagrams
- **Filters** = {NoCrossedDiagrams} is used to drop diagrams which only differ by a permutation of the external fields.

For instance, the **PreSARAH** input to calculate the coefficient of the $(\bar{\ell}\Gamma\ell)(\bar{d}\Gamma'd)$ operator reads

```

1 NameProcess="2L2d ";
2 ConsideredProcess = "4Fermion ";
3 ExternalFields={{ChargedLepton , bar [ ChargedLepton ] ,
4                 DownQuark , bar [ DownQuark ] }};
5
6 FermionOrderExternal={2,1,4,3};
7 NeglectMasses={1,2,3,4};
8
9
10 AllOperators={
11   (* scalar operators*)
12   {Ol1ddSSL,Op[7].Op[7]} ,
13   {Ol1ddSRR,Op[6].Op[6]} ,
14   {Ol1ddSRL,Op[6].Op[7]} ,
15   {Ol1ddSLR,Op[7].Op[6]} ,
16
17   (* vector operators*)
18   {Ol1ddVRR,Op[7,Lor[1]].Op[7,Lor[1]]} ,

```

```

19 {O1l1ddVLL,Op[6,Lor[1]].Op[6,Lor[1]]},
20 {O1l1ddVRL,Op[7,Lor[1]].Op[6,Lor[1]]},
21 {O1l1ddVLR,Op[6,Lor[1]].Op[7,Lor[1]]},
22
23 (* tensor operators*)
24 {O1l1ddTLL,Op[-7,Lor[1],Lor[2]].Op[-7,Lor[1],Lor[2]]},
25 {O1l1ddTLR,Op[-7,Lor[1],Lor[2]].Op[-6,Lor[1],Lor[2]]},
26 {O1l1ddTRL,Op[-6,Lor[1],Lor[2]].Op[-7,Lor[1],Lor[2]]},
27 {O1l1ddTRR,Op[-6,Lor[1],Lor[2]].Op[-6,Lor[1],Lor[2]]}
28 };
29
30 CombinationGenerations = {{2,1,1,1},{3,1,1,1},{3,2,1,1},
31                             {2,1,2,2},{3,1,2,2},{3,2,2,2}};
32
33 Filters = {};

```

Here, we neglect all external masses in the operators (**NeglectMasses**={1,2,3,4}), and the different coefficients of the scalar operators ($\bar{\ell}P_X\ell)(\bar{d}P_Yd)$ are called **O1l1ddSXY**, the ones for the vector operators ($\bar{\ell}P_X\gamma_\mu\ell)(\bar{d}P_Y\gamma^\mu d)$ are called **O1l1ddVYX** and the ones for the tensor operators ($\bar{\ell}P_X\sigma_{\mu\nu}\ell)(\bar{d}\sigma^{\mu\nu}P_Yd)$ **O1l1ddTYX**, with X,Y=L,R. Notice that **FormCalc** returns the results in form of $P_X\gamma_\mu$ while in the literature the order $\gamma_\mu P_X$ is often used. Finally, **SPheno** will not calculate all possible combinations of external states, but only some specific cases: $\mu edd, \tau edd, \tau\mu dd, \mu ess, \tau ess, \tau\mu ss$ ⁸.

The input file to calculate the coefficients of the $\ell-\ell-Z$ operators ($\bar{\ell}\gamma_\mu P_{L,R}\ell)Z^\mu$ and ($\bar{\ell}p_\mu P_{L,R}\gamma_\mu\ell)Z^\mu$ is

```

1 NameProcess="Z2l ";
2
3 ConsideredProcess = "2Fermion1Vector ";
4 FermionOrderExternal={1,2};
5 NeglectMasses={1,2};
6
7
8 ExternalFields= {ChargedLepton,bar[ChargedLepton],Zboson};
9 CombinationGenerations = {{1,2},{1,3},{2,3}};
10
11
12 AllOperators={
13   {OZ2lSL,Op[7]}, {OZ2lSR,Op[6]},
14   {OZ2lVL,Op[7,ec[3]]}, {OZ2lVR,Op[6,ec[3]]}
15 };
16
17 OutputFile = "Z2l.m";
18
19 Filters = {};

```

Note that **ExternalFields** must contain first the involved fermions and the boson at the end. Furthermore, in the case of processes involving scalars one can define

```

1 ExternalFields= {ChargedLepton,bar[ChargedLepton],ScalarHiggs};
2 CombinationGenerations = {{1,2,ALL},{1,3,ALL},{2,3,ALL}};

```

In this case the operators for all Higgs states present in the considered model will be computed.

⁸Here we used d for the first generation of down-type quarks while in the rest of this manual it is used to summarize all three families.

5.3 Operators with massless gauge bosons

We have to add a few more remarks concerning 2-fermion-1-boson operators with massless gauge bosons since those are treated in a special way. It is common for these operators to include terms in the amplitude which are proportional to the external masses. Therefore, if one proceeds in the usual way and neglects the external momenta, some inconsistencies would be obtained. For this reason, a special treatment is in order. In **PreSARAH**, when one uses

```
1 ConsideredProcess = "2Fermion1Vector";
2 FermionOrderExternal={1,2};
3 NeglectMasses={3};
```

the dependence on the two fermion masses is neglected in the resulting Passarino-Veltman integrals but terms proportional to m_{f_1} and m_{f_2} are kept. This solves the aforementioned potential inconsistencies.

Furthermore, for the dipole operators, defined by

```
1 {DipoleL, Op[6] Pair[ec[3], k[1]]},
2 {DipoleR, Op[7] Pair[ec[3], k[1]]},
```

we are using the results obtained by **FeynArts** and **FormCalc** and have implemented all special cases for the involved loop integrals ($C_0, C_{00}, C_1, C_2, C_{11}, C_{12}, C_{22}$) with identical or vanishing internal masses in **SPheno**. This guarantees the numerical stability of the results⁹.

The monopole operators of the form $q^2(\bar{f}\gamma_\mu f)V^\mu$ are only non-zero for off-shell external gauge bosons, while **PreSARAH** always treats all fields as on-shell. Because of this, and to stabilize the numerical evaluation later on, these operators are treated differently to all other operators: the coefficients are not calculated by **FeynArts** and **FormCalc** but instead we have included the generic expressions in **PreSARAH** using a special set of loop functions in **SPheno**. In these loop functions the resulting Passarino-Veltman integrals are already combined, leading to well-known expressions in the literature, see [42, 56]. They have been cross-checked with the package **Peng4BSM@LO** [43]. To get the coefficients for the monopole operators, these have to be given always in the form

```
1 {MonopoleL, Op[6, ec[3]] Pair[k[3], k[3]]},
2 {MonopoleR, Op[7, ec[3]] Pair[k[3], k[3]]}
```

in the input of **PreSARAH**.

5.4 Combination and normalization of operators

The user can define new operators as combination of existing operators. For this purpose wrapper files containing the definition of the operators can be included in the **FlavorKit** directories. These files have to begin with **ProcessWrapper = True;**. This function is also used by **PreSARAH** in the case of 4-fermion operators: for these operators the contributions stemming from tree-level, box- and penguin- diagrams are saved separately and summed up at the end. Thus, the wrapper code for the 4-lepton operators written by **PreSARAH** reads

```
1 ProcessWrapper = True;
2 NameProcess = "4L"
3 ExternalFields = {ChargedLepton, bar[ChargedLepton], ChargedLepton, 
4   ↵ bar[ChargedLepton]};
5 SumContributionsOperators["4L"] = {
6   {O4ISLL, BO4ISLL + PSO4ISLL + PVO4ISLL + TSO4ISLL + TVO4ISLL},
7   {O4ISRR, BO4ISRR + PSO4ISRR + PVO4ISRR + TSO4ISRR + TVO4ISRR},
8   ...
9   };

```

⁹We note that the coefficients for the operators defined above ($\bar{f}\gamma_\mu f V^\mu$) are by a factor of 2 (4) larger than the coefficients of the standard definition for the dipole operators $\bar{f}\sigma_{\mu\nu}P_L f q^\nu V^\mu$ ($\bar{f}\sigma_{\mu\nu}P_L f F^{\mu\nu}$).

It is also possible to use these wrapper files to change the normalization of the operators. We have made use of this functionality for the operators with external photons and gluons to match the standard definition used in literature: it is common to write these operators as $e m_f (\bar{f} \sigma_{\mu\nu} f) F^{\mu\nu}$, i.e. with the electric coupling (or strong coupling for gluon operators) and fermion mass factored out. This is done with the files `Photon_wrapper.m` and `Gluon_wrapper.m`, included in the FlavorKit directory of SARAH:

```

1 ProcessWrapper = True;
2 NameProcess = "Gamma2Q"
3 ExternalFields = {bar[BottomQuark], BottomQuark, Photon};
4
5 SumContributionsOperators["Gamma2Q"] = {
6   {CC7, OA2qSL},
7   {CC7p, OA2qSR}
8 };
9
10 NormalizationOperators["Gamma2Q"] = {
11   "CC7(3,:) = 0.25_dp*CC7(3,:)/sqrt(Alpha_160*4*Pi)/mf_d_160(3)",
12   "CC7p(3,:) = 0.25_dp*CC7p(3,:)/sqrt(Alpha_160*4*Pi)/mf_d_160(3)",
13
14   "CC7SM(3,:) = 0.25_dp*CC7SM(3,:)/sqrt(Alpha_160*4*Pi)/mf_d_160(3)",
15   "CC7pSM(3,:) = 0.25_dp*CC7pSM(3,:)/sqrt(Alpha_160*4*Pi)/mf_d_160(3)"
16 };

```

First, the coefficients `OA2qSL` and `OA2qSR` derived with `PreSARAH` are matched to the new coefficients `CC7` and `CC7p`. The same matching is automatically applied also to the SM coefficients `OA2qSLSM` and `OA2qSRSM`. In a second step, these operators are re-normalized to the standard definition of the Wilson coefficients C_7 and C'_7 . The initial coefficients `OA2qSR`, `OA2qSL` are not discarded, but co-exist besides `CC7`, `CC7p`. Thus, the user can choose in the implementation of the observables which operators are more suitable for his purposes.

6 Validation

The validation of the `FlavorKit` results happened in three steps:

1. **Agreement with SM results:** we checked that the SM prediction for the observables agree with the results given in literature
2. **Independence of scale in loop function:** the loop integrals for two and three point functions (B_i, C_i) depend on the renormalization scale Q . However, this dependence has to drop out for a given process at leading order. We checked numerically that the sum of all diagrams is indeed independent of the choice of Q .
3. **Comparison with other tools:** as we have pointed out in the introduction, there are several public tools which calculate flavor observables mostly in the context of the MSSM. We did a detailed comparison with these tools using the `SPheno` code produced by `SARAH` for the MSSM. Some results are presented in the following.

We have compared the `FlavorKit` results using `SARAH4.2.0` and `SPheno3.3.0` with

- `superiso 3.3`
- `SUSY_Flavor 1` and `2.1`
- `MicrOmegas 3.6.7`
- `SPheno 3.3.0`
- a `SPheno` version produced by `SARAH 4.1.0` without the `FlavorKit` functionality

Since these codes often use different values for the hadronic parameters and calculate the flavor observables at different loop levels, we are not going to compare the absolute numbers obtained by these tools. Instead, we compare the results normalized to the SM prediction of each code and thus define, for an observable X , the ratio

$$R(X) = \frac{X^{MSSM}}{X^{SM}}. \quad (6)$$

X^{SM} is obtained by taking the value of X calculated by each code in the limit of a very heavy SUSY spectrum. As test case we have used the CMSSM. The dependence of a set of flavor observables as function of m_0 is shown in Fig. 7 and as function of $M_{1/2}$ in Fig. 8.

We see that all codes show in general the same dependence. However, it is also obvious that the lines are not on top of each other but differences are present. These differences are based on the treatment of the resummation of the bottom Yukawa couplings, the different order at which SM and SUSY contributions are implemented, the different handling of the Weinberg angle, and the different level at which the RGE running is taken into account by the tools. Even if a detailed discussion of the differences of all codes might be very interesting it is, of course, far beyond the scope of this paper and would require a combined effort. The important point is that the results of **FlavorKit** agree with the codes specialized for the MSSM to the same level as those codes agree among each other. Since the **FlavorKit** results for all observables are based on the same generic routines it might be even more trustworthy than human implementations of the lengthy expressions needed to calculate these observables because it is less error prone. Of course, known 2-loop corrections for the MSSM which are implemented in other tools are missing.

Finally, it is well known that the process $B_{s,d}^0 \rightarrow \ell\bar{\ell}$ has a strong dependence on the value of $\tan\beta$. We show in Fig. 9 that this is reproduced by all codes.

7 Conclusion

We have presented **FlavorKit**, a new setup for the calculation of flavor observables for a wide range of BSM models. Generic expressions for the Wilson coefficients are derived with **PreSARAH**, a **Mathematica** package that makes use of **FeynArts** and **FormCalc**. The output of **PreSARAH** is then passed to **SARAH**, which generates the **Fortran** code that allows to calculate numerically the values of these Wilson coefficients with **SPheno**. The observables are derived by providing the corresponding pieces of **Fortran** code to **SARAH**, which incorporates them into the **SPheno** output. We made use of this code chain to fully implement a large set of important flavor observables in **SARAH** and **SPheno**. In fact, due the simplicity of this kit, the user can easily extend the list with his own observables and operators. In conclusion, **FlavorKit** allows the user to easily obtain analytical and numerical results for flavor observables in the BSM model of his choice.

Acknowledgments

We thank Asmaa Abada, Martin Hirsch, Farvah Mahmoudi, Manuel E. Krauss, Kilian Nickel, Ben O’Leary and Cédric Weiland for helpful discussions. FS is supported by the BMBF PT DESY Verbundprojekt 05H2013-THEORIE ‘Vergleich von LHC-Daten mit supersymmetrischen Modellen’. WP is supported by the DFG, project No. PO-1337/3-1. AV is partially supported by the EXPL/FIS-NUC/0460/2013 project financed by the Portuguese FCT.

A: Lagrangian

In this section we present our notation and conventions for the operators (and their corresponding Wilson coefficients) implemented in **PreSARAH**. Although a more complete list of flavor violating operators can be built, we will concentrate on those implemented in **PreSARAH**. If necessary, the user can extend it by adding his/her own operators.

The interaction Lagrangian relevant for flavor violating processes can be written as

$$\mathcal{L}_{FV} = \mathcal{L}_{LFV} + \mathcal{L}_{QFV}. \quad (\text{A.1})$$

The first piece contains the operators that can trigger lepton flavor violation whereas the second piece contains the operators responsible for quark flavor violation.

The general Lagrangian relevant for lepton flavor violation can be written as

$$\mathcal{L}_{LFV} = \mathcal{L}_{\ell\ell\gamma} + \mathcal{L}_{\ell\ell Z} + \mathcal{L}_{\ell\ell h} + \mathcal{L}_{4\ell} + \mathcal{L}_{2\ell 2q}. \quad (\text{A.2})$$

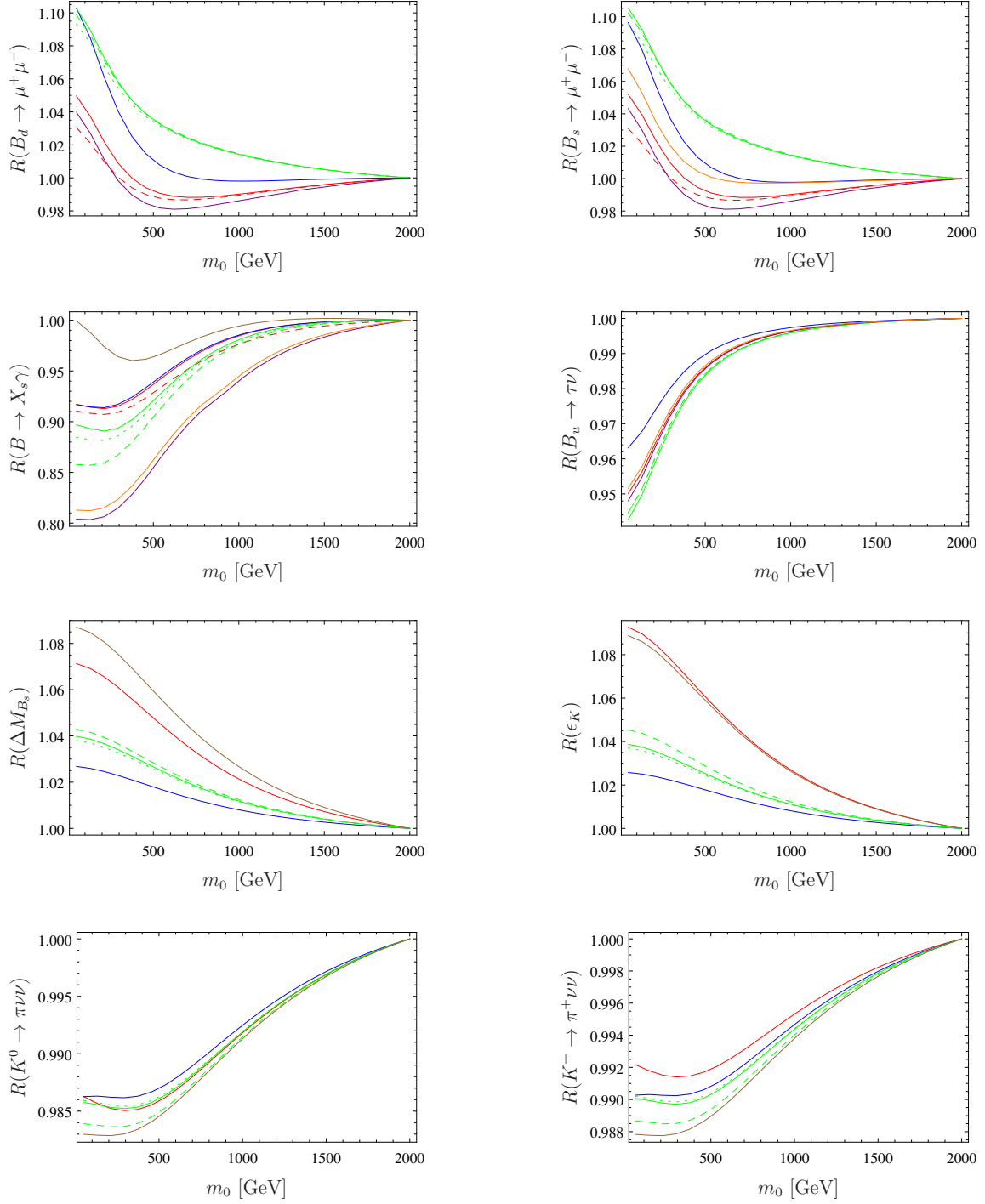


Fig. 7 Comparison of the results for $\text{BR}(B_{s,d}^0 \rightarrow \mu\mu)$, $\text{BR}(\bar{B} \rightarrow X_s \gamma)$, $\text{BR}(B \rightarrow \tau\nu)$, ΔM_{B_s} , ϵ_K , $\text{BR}(K_L \rightarrow \pi^0 \nu\bar{\nu})$, $\text{BR}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$ as a function of m_0 using the FlavorKit (red), superiso (purple), SUSY_Flavor 1 (brown), SUSY_Flavor 2 (green), SPheno (blue), MicrOmegas (orange) and the old implementation in SARAH (red dashed). The three lines for SUSY_Flavor 2 correspond to different options of the chiral resummation. We used $M_{1/2} = 200$ GeV, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$.

The first term contains the $\ell - \ell - \gamma$ interaction, given by

$$\mathcal{L}_{\ell\ell\gamma} = e \bar{\ell}_\beta \left[\gamma^\mu \left(K_1^L P_L + K_1^R P_R \right) + i m_{\ell_\alpha} \sigma^{\mu\nu} q_\nu \left(K_2^L P_L + K_2^R P_R \right) \right] \ell_\alpha A_\mu + \text{h.c.} \quad (\text{A.3})$$

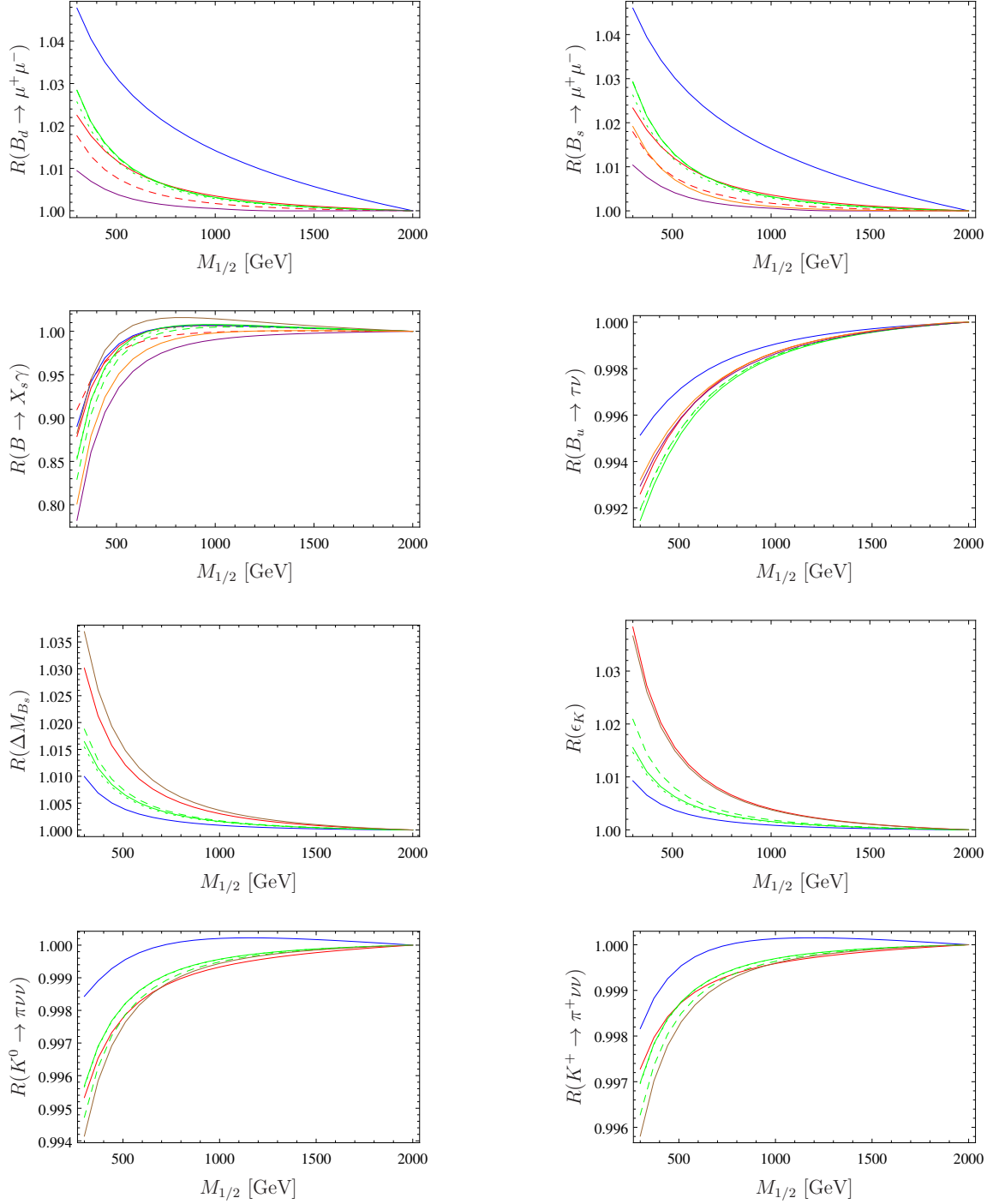


Fig. 8 Comparison of the results for different flavor observables as function of $M_{1/2}$. The color code is the same as in Fig. 7. We used $m_0 = 500$ GeV, $A_0 = -1000$ GeV, $\tan \beta = 10$, $\mu > 0$.

Here e is the electric charge, q the photon momentum, $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ are the usual chirality projectors and $\ell_{\alpha,\beta}$ denote the lepton flavors. For practical reasons, we will always consider the photonic contributions independently, and we will not include them in other vector operators. On the contrary, the Z - and Higgs boson contributions will be included whenever possible. Therefore, the $\ell - \ell - Z$ and $\ell - \ell - h$ interaction Lagrangians

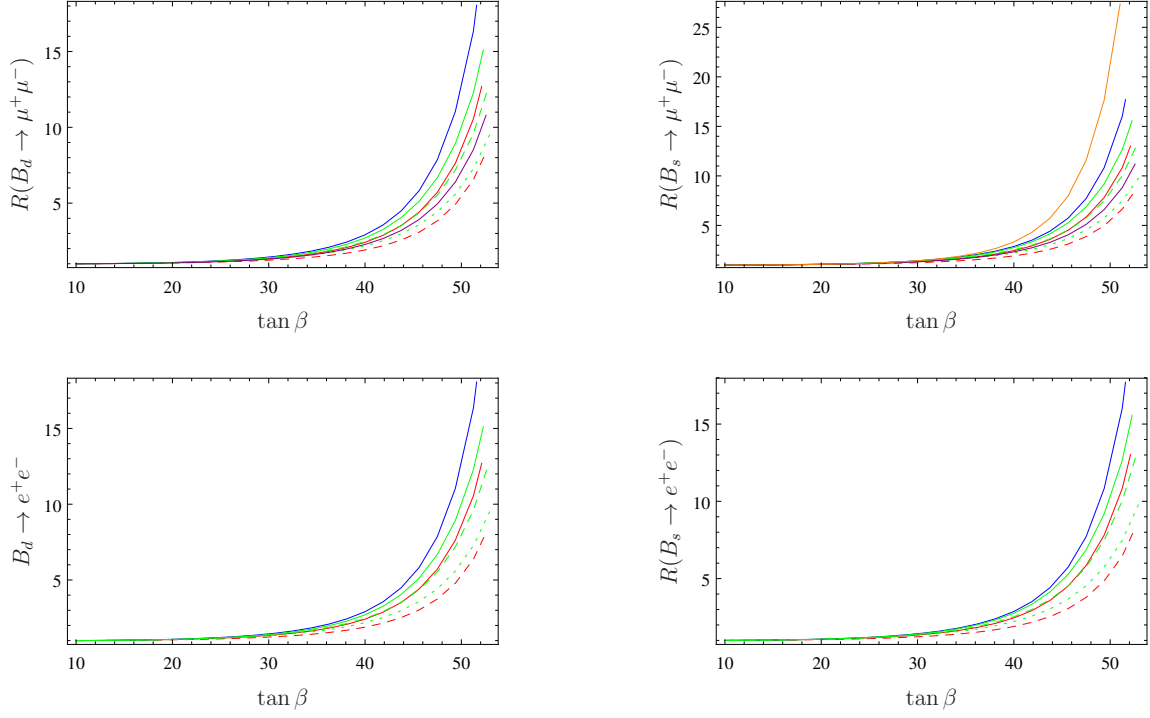


Fig. 9 Comparison of $\text{BR}(B_{s,d}^0 \rightarrow \mu\mu)$ (first row) and $\text{BR}(B_{s,d}^0 \rightarrow ee)$ (second row) as function of $\tan\beta$. The color code is the same as in Fig. 7. We used $m_0 = M_{1/2} = 500$ GeV, $A_0 = 0$, $\mu > 0$.

will only be used for observables involving real Z- and Higgs bosons. These two Lagrangians can be written as

$$\mathcal{L}_{\ell\ell Z} = \bar{\ell}_\beta \left[\gamma^\mu \left(R_1^L P_L + R_1^R P_R \right) + p^\mu \left(R_2^L P_L + R_2^R P_R \right) \right] \ell_\alpha Z_\mu, \quad (\text{A.4})$$

where p is the ℓ_β 4-momentum, and

$$\mathcal{L}_{\ell\ell h} = \bar{\ell}_\beta (S_L P_L + S_R P_R) \ell_\alpha h. \quad (\text{A.5})$$

The general 4ℓ 4-fermion interaction Lagrangian can be written as

$$\mathcal{L}_{4\ell} = \sum_{\substack{I=S,V,T \\ X,Y=L,R}} A_{XY}^I \bar{\ell}_\beta \Gamma_I P_X \ell_\alpha \bar{\ell}_\delta \Gamma_I P_Y \ell_\gamma + \text{h.c.}, \quad (\text{A.6})$$

where $\ell_{\alpha,\beta,\gamma,\delta}$ denote the lepton flavors and $\Gamma_S = 1$, $\Gamma_V = \gamma_\mu$ and $\Gamma_T = \sigma_{\mu\nu}$. We omit flavor indices in the Wilson coefficients for the sake of clarity. This Lagrangian contains the most general form compatible with Lorentz invariance. The Wilson coefficients A_{LR}^S and A_{RL}^S were included in [57], but absent in [42, 58]. As previously stated, the coefficients in Eq.(A.6) do not include photonic contributions, but they include Z-boson and scalar ones. Finally, the general $2\ell 2q$ four fermion interaction Lagrangian at the quark level is given by

$$\mathcal{L}_{2\ell 2q} = \mathcal{L}_{2\ell 2d} + \mathcal{L}_{2\ell 2u} \quad (\text{A.7})$$

where

$$\mathcal{L}_{2\ell 2d} = \sum_{\substack{I=S,V,T \\ X,Y=L,R}} B_{XY}^I \bar{\ell}_\beta \Gamma_I P_X \ell_\alpha \bar{d}_\gamma \Gamma_I P_Y d_\gamma + \text{h.c.} \quad (\text{A.8})$$

$$\mathcal{L}_{2\ell 2u} = \mathcal{L}_{2\ell 2d}|_{d \rightarrow u, B \rightarrow C}. \quad (\text{A.9})$$

Here d_γ denotes the d-quark flavor.

Let us now consider the Lagrangian relevant for quark flavor violation. This can be written as

$$\mathcal{L}_{\text{QFV}} = \mathcal{L}_{qq\gamma} + \mathcal{L}_{qqg} + \mathcal{L}_{4d} + \mathcal{L}_{2d2l} + \mathcal{L}_{2d2\nu} + \mathcal{L}_{dul\nu} + \mathcal{L}_{ddH}. \quad (\text{A.10})$$

The first two terms correspond to operators that couple quark bilinears to massless gauge bosons. These are

$$\mathcal{L}_{qq\gamma} = e \left[\bar{d}_\beta \sigma_{\mu\nu} \left(m_{d_\beta} Q_1^L P_L + m_{d_\alpha} Q_1^R P_R \right) d_\alpha \right] F^{\mu\nu} \quad (\text{A.11})$$

$$\mathcal{L}_{qqg} = g_s \left[\bar{d}_\beta \sigma_{\mu\nu} \left(m_{d_\beta} Q_2^L P_L + m_{d_\alpha} Q_2^R P_R \right) T^a d_\alpha \right] G_a^{\mu\nu}. \quad (\text{A.12})$$

Here T^a are $SU(3)$ matrices. The Wilson coefficients $Q_{1,2}^{L,R}$ can be easily related to the usual $C_{7,8}^{(\prime)}$ coefficients, sometimes normalized with an additional $\frac{1}{16\pi^2}$ factor. The $4d$ four fermion interaction Lagrangian can be written as

$$\mathcal{L}_{4d} = \sum_{\substack{I=S,V,T \\ X,Y=L,R}} D_{XY}^I \bar{d}_\beta \Gamma_I P_X d_\alpha \bar{d}_\delta \Gamma_I P_Y d_\gamma + \text{h.c.}, \quad (\text{A.13})$$

where $d_{\alpha,\beta,\gamma,\delta}$ denote the lepton flavors. Again, we omit flavor indices in the Wilson coefficients for the sake of clarity. The $2d2l$ four fermion interaction Lagrangian is given by

$$\mathcal{L}_{2d2l} = \sum_{\substack{I=S,V,T \\ X,Y=L,R}} E_{XY}^I \bar{d}_\beta \Gamma_I P_X d_\alpha \bar{\ell}_\gamma \Gamma_I P_Y \ell_\gamma + \text{h.c.}. \quad (\text{A.14})$$

Here ℓ_γ denotes the lepton flavor. \mathcal{L}_{2d2l} should not be confused with \mathcal{L}_{2l2d} . In the former case one has QFV operators, whereas in the latter one has LFV operators. This distinction has been made for practical reasons. The $2d2\nu$ and $dul\nu$ terms of the QFV Lagrangian are

$$\mathcal{L}_{2d2\nu} = \sum_{X,Y=L,R} F_{XY}^V \bar{d}_\beta \gamma_\mu P_X d_\alpha \bar{\nu}_\gamma \gamma^\mu P_Y \nu_\gamma + \text{h.c.} \quad (\text{A.15})$$

$$\mathcal{L}_{dul\nu} = \sum_{\substack{I=S,V \\ X,Y=L,R}} G_{XY}^I \bar{d}_\beta \Gamma_I P_X u_\alpha \bar{\ell}_\gamma \Gamma_I P_Y \nu_\gamma + \text{h.c.}. \quad (\text{A.16})$$

Note that we have not introduced scalar or tensor $2d2\nu$ operators, nor tensor $dul\nu$ ones, and that lepton flavor (denoted by the index γ) is conserved in these operators. Finally, we have also included a term in the Lagrangian accounting for operators of the type $(\bar{d}\Gamma d)S$ and $(\bar{d}\Gamma d)P$, where S (P) is a virtual ¹⁰ scalar (pseudoscalar) state. This piece can be written as

$$\mathcal{L}_{ddH} = \bar{d}_\beta \left(H_L^S P_L + H_R^S P_R \right) d_\alpha S + \bar{d}_\beta \left(H_L^P P_L + H_R^P P_R \right) d_\alpha P. \quad (\text{A.17})$$

B: Operators available by default in the SPHeno output of SARAH

The operators presented in Appendix A have been implemented by using the results of PreSARAH in SARAH. Those are exported to SPHeno. We give in the following the list of all internal names for these operators, which can be used in the calculation of new flavor observables.

¹⁰We would like to emphasize that our implementation of these operators is only valid for virtual scalars and pseudoscalars. They have been introduced in order to provide the 1-loop vertices necessary for the computation of the double penguin contributions to ΔM_{B_q} . Therefore, they are not valid for observables in which the scalar or pseudoscalar states are real particles.

B.1: 2-Fermion-1-Boson operators

These operators are arrays with either two or three elements. While operators involving vector bosons have always dimension 3×3 , those with scalars have dimension $3 \times 3 \times n_g$. n_g is the number of generations of the considered scalar and for $n_g = 1$ the last index is dropped.

$$(\bar{d}_\beta \sigma_{\mu\nu} \Gamma d_\alpha) F^{\mu\nu} \text{ and } (\bar{d}_\beta \sigma_{\mu\nu} \Gamma d_\alpha) G^{\mu\nu}$$

Variable	Operator	Name	Variable	Operator	Name
CC7	$em_{d_\beta}(\bar{d}_\beta \sigma_{\mu\nu} P_L d_\alpha) F^{\mu\nu}$	Q_1^L	CC8	$g_s m_{d_\beta}(\bar{d}_\beta \sigma_{\mu\nu} P_L d_\alpha) G^{\mu\nu}$	Q_2^L
CC7p	$em_{d_\alpha}(\bar{d}_\beta \sigma_{\mu\nu} P_R d_\alpha) F^{\mu\nu}$	Q_1^R	CC8p	$g_s m_{d_\alpha}(\bar{d}_\beta \sigma_{\mu\nu} P_R d_\alpha) G^{\mu\nu}$	Q_2^R

These operators are derived by **PreSARAH** with the following input files

Listing 1 PhotonQQp.m

```

1 NameProcess="Gamma2Q";
2
3 ConsideredProcess = "2Fermion1Vector";
4 FermionOrderExternal={1,2};
5 NeglectMasses={3};
6
7
8 ExternalFields= {bar[BottomQuark], BottomQuark, Photon};
9 CombinationGenerations = {{3,2}};
10
11
12 AllOperators={
13   {OA2qSL,Op[7] Pair[ec[3],k[1]]},
14   {OA2qSR,Op[6] Pair[ec[3],k[1]]},
15   {OA2qVL,Op[7,ec[3]]},
16   {OA2qVR,Op[6,ec[3]]}
17 };
18
19 OutputFile = "Gamma2Q.m";
20
21 Filters = {};
```

Listing 2 GluonQQp.m

```

1 NameProcess="Gluon2Q";
2
3 ConsideredProcess = "2Fermion1Vector";
4 FermionOrderExternal={1,2};
5 NeglectMasses={3};
6
7
8 ExternalFields= {bar[BottomQuark], BottomQuark, Gluon};
9 CombinationGenerations = {{3,2}};
10
11
12 AllOperators={
13   {OG2qSL,Op[7] Pair[ec[3],k[1]]},
14   {OG2qSR,Op[6] Pair[ec[3],k[1]]}
15 };
16
17 OutputFile = "Gluon2Q.m";
18
19 Filters = {};
```

The normalization is changed to match the standard definitions by

Listing 3 Photon_wrapper_QFV.m

```

1 ProcessWrapper = True;
```

```

1 NameProcess = "Gamma2Q"
2 ExternalFields = {bar[BottomQuark], BottomQuark, Photon};
3
4
5 SumContributionsOperators["Gamma2Q"] = {
6 {CC7, OA2qSL},
7 {CC7p, OA2qSR}
8 };
9
10 NormalizationOperators["Gamma2Q"] = {
11 "CC7(2,:) = 0.25_dp*CC7(2,:)/sqrt(Alpha_160*4*Pi)/MFd(2)",
12 "CC7(3,:) = 0.25_dp*CC7(3,:)/sqrt(Alpha_160*4*Pi)/MFd(3)",
13 "CC7p(2,:) = 0.25_dp*CC7p(2,:)/sqrt(Alpha_160*4*Pi)/MFd(2)",
14 "CC7p(3,:) = 0.25_dp*CC7p(3,:)/sqrt(Alpha_160*4*Pi)/MFd(3)",
15
16 "CC7SM(2,:) = 0.25_dp*CC7SM(2,:)/sqrt(Alpha_160*4*Pi)/MFd(2)",
17 "CC7SM(3,:) = 0.25_dp*CC7SM(3,:)/sqrt(Alpha_160*4*Pi)/MFd(3)",
18 "CC7pSM(2,:) = 0.25_dp*CC7pSM(2,:)/sqrt(Alpha_160*4*Pi)/MFd(2)",
19 "CC7pSM(3,:) = 0.25_dp*CC7pSM(3,:)/sqrt(Alpha_160*4*Pi)/MFd(3)"
20 };

```

Listing 4 Gluon_wrapper.m

```

1 ProcessWrapper = True;
2 NameProcess = "Gluon2Q"
3 ExternalFields = {bar[BottomQuark], BottomQuark, Gluon};
4
5 SumContributionsOperators["Gluon2Q"] = {
6 {CC8, OG2qSL},
7 {CC8p, OG2qSR}};
8
9 NormalizationOperators["Gluon2Q"] = {
10 "CC8(2,:) = 0.25_dp*CC8(2,:)/sqrt(AlphaS_160*4*Pi)/MFd(2)",
11 "CC8(3,:) = 0.25_dp*CC8(3,:)/sqrt(AlphaS_160*4*Pi)/MFd(3)",
12 "CC8p(2,:) = 0.25_dp*CC8p(2,:)/sqrt(AlphaS_160*4*Pi)/MFd(2)",
13 "CC8p(3,:) = 0.25_dp*CC8p(3,:)/sqrt(AlphaS_160*4*Pi)/MFd(3)",
14
15 "CC8SM(2,:) = 0.25_dp*CC8SM(2,:)/sqrt(AlphaS_160*4*Pi)/MFd(2)",
16 "CC8SM(3,:) = 0.25_dp*CC8SM(3,:)/sqrt(AlphaS_160*4*Pi)/MFd(3)",
17 "CC8pSM(2,:) = 0.25_dp*CC8pSM(2,:)/sqrt(AlphaS_160*4*Pi)/MFd(2)",
18 "CC8pSM(3,:) = 0.25_dp*CC8pSM(3,:)/sqrt(AlphaS_160*4*Pi)/MFd(3)"
19 };
20

```

$$\bar{\ell}_\beta (q^2 \gamma^\mu + i m_{\ell_\alpha} \sigma^{\mu\nu} q_\nu) \ell_\alpha A_\mu$$

Variable	Operator	Name	Variable	Operator	Name
K2L	$em_{\ell_\alpha} (\bar{\ell}_\beta \sigma_{\mu\nu} P_L \ell_\alpha) q^\nu A^\mu$	K_2^L	K1L	$q^2 (\bar{\ell}_\beta \gamma_\mu P_L \ell_\alpha) A^\mu$	K_1^L
K2R	$em_{\ell_\alpha} (\bar{\ell}_\beta \sigma_{\mu\nu} P_R \ell_\alpha) q^\nu A^\mu$	K_2^R	K1R	$q^2 (\bar{\ell}_\beta \gamma_\mu P_R \ell_\alpha) A^\mu$	K_1^R

These operators are derived by **PreSARAH** with the following input files

Listing 5 PhotonLLp.m

```

1 NameProcess="Gamma2l";
2
3 ConsideredProcess = "2Fermion1Vector";
4 FermionOrderExternal={1,2};
5 NeglectMasses={3};
6
7
8 ExternalFields= {bar[ChargedLepton], ChargedLepton, Photon};
9 CombinationGenerations = {{2,1},{3,1},{3,2}};
10
11
12 AllOperators={

```



```

13 {OA2ISL,Op[6] Pair[ec[3],k[1]]},
14 {OA2ISR,Op[7] Pair[ec[3],k[1]]},
15 {OA1L,Op[6,ec[3]] Pair[k[3],k[3]]},
16 {OA1R,Op[7,ec[3]] Pair[k[3],k[3]]}
17 };
18
19 OutputFile = "Gamma2l.m";
20
21 Filters = {};

```

The normalization is changed to match the standard definitions by

Listing 6 Photon_wrapper_LFV.m

```

1 ProcessWrapper = True;
2 NameProcess = "Gamma2l"
3 ExternalFields = {bar[ChargedLepton], ChargedLepton, Photon};
4
5 SumContributionsOperators["Gamma2l"] = {
6 {K1L, OA1L},
7 {K1R, OA1R},
8 {K2L, OA2ISL},
9 {K2R, OA2ISR}};
10
11 NormalizationOperators["Gamma2l"] = {
12 "K1L = K1L/sqrt(Alpha_MZ*4*Pi)",
13 "K1R = K1R/sqrt(Alpha_MZ*4*Pi)",
14 "K2L(2,:) = -0.5_dp*K2L(2,:)/sqrt(Alpha_MZ*4*Pi)/MFe(2)",
15 "K2L(3,:) = -0.5_dp*K2L(3,:)/sqrt(Alpha_MZ*4*Pi)/MFe(3)",
16 "K2R(2,:) = -0.5_dp*K2R(2,:)/sqrt(Alpha_MZ*4*Pi)/MFe(2)",
17 "K2R(3,:) = -0.5_dp*K2R(3,:)/sqrt(Alpha_MZ*4*Pi)/MFe(3)"
18 };

```

$$(\bar{\ell}\Gamma\ell)Z$$

Variable	Operator	Name	Variable	Operator	Name
OZ2IVL	$(\bar{\ell}\gamma^\mu P_L\ell)Z_\mu$	R_1^L	OZ2ISL	$(\bar{\ell}p^\mu P_L\ell)Z_\mu$	R_2^L
OZ2IVR	$(\bar{\ell}\gamma^\mu P_R\ell)Z_\mu$	R_1^R	OZ2ISR	$(\bar{\ell}p^\mu P_R\ell)Z_\mu$	R_2^R

In the following we omit flavor indices for the sake of simplicity. These operators are derived by **PreSARAH** with the following input files

Listing 7 Z2l.m

```

1 NameProcess="Z2l";
2
3 ConsideredProcess = "2Fermion1Vector";
4 FermionOrderExternal={1,2};
5 NeglectMasses={1,2};
6
7
8 ExternalFields= {ChargedLepton,bar[ChargedLepton],Zboson};
9 CombinationGenerations = {{1,2},{1,3},{2,3}};
10
11
12 AllOperators={
13 {OZ2ISL,Op[7] Pair[ec[3],k[1]]}, {OZ2ISR,Op[6] Pair[ec[3],k[1]]},
14 {OZ2IVL,Op[7,ec[3]]}, {OZ2IVR,Op[6,ec[3]]}
15 };
16
17 OutputFile = "Z2l.m";
18
19 Filters = {};

```

$$(\bar{\ell}\Gamma\ell)h$$

Variable	Operator	Name	Variable	Operator	Name
OH2lSL	$\bar{\ell} P_L \ell h$	S_L	OH2lSR	$\bar{\ell} P_R \ell h$	S_R

These operators are derived by **PreSARAH** with the following input files

Listing 8 H2l.m

```

1 NameProcess="H2l";
2
3 ConsideredProcess = "2Fermion1Scalar";
4 FermionOrderExternal={1,2};
5 NeglectMasses={1,2};
6
7
8 ExternalFields= {ChargedLepton, bar [ChargedLepton], HiggsBoson};
9 CombinationGenerations = {{1,2,ALL},{1,3,ALL},{2,3,ALL}};
10
11
12 AllOperators={{OH2lSL,Op[7]},
13               {OH2lSR,Op[6]}}
14 };
15
16 OutputFile = "H2l.m";
17
18 Filters = {};
```

$(\bar{d}\Gamma d)S$ and $(\bar{d}\Gamma d)P$

Variable	Operator	Name	Variable	Operator	Name
OH2qSL	$\bar{d} P_L d S$	H_L^S	OH2qSR	$\bar{d} P_R d S$	H_R^S
OAh2qSL	$\bar{d} P_L d P$	H_L^P	OAh2qSR	$\bar{d} P_R d P$	H_R^P

These auxiliary ¹¹ operators are derived by **PreSARAH** with the following input files

Listing 9 H2q.m

```

1 NameProcess="H2q";
2
3 (* operators needed for double penguins with internal scalars *)
4 (* we neglect therefore the mass of the scalar in the loop functions *)
5 (* and treat it as massless *)
6
7 ConsideredProcess = "2Fermion1Scalar";
8 FermionOrderExternal={2,1};
9 NeglectMasses={3};
10
11
12 ExternalFields= {DownQuark, bar [DownQuark], HiggsBoson};
13 CombinationGenerations = {{2,1,ALL},{3,1,ALL},{3,2,ALL}};
14
15
16 AllOperators={{OH2qSL,Op[7]},
17               {OH2qSR,Op[6]}}
18 };
19
20 OutputFile = "H2q.m";
21
22 Filters = {};
```

¹¹The $(\bar{d}\Gamma d)S$ and $(\bar{d}\Gamma d)P$ operators have been introduced to compute double penguin corrections to ΔM_{B_q} , where S and P appear as intermediate (virtual) particles. They should not be used in processes where the scalar or pseudoscalar states are real particles because the loop functions are calculated with vanishing external momenta.

Listing 10 A2q.m

```

1 NameProcess="A2q";
2
3 (* operators needed for double penguins with internal scalars *)
4 (* we neglect therefore the mass of the scalar in the loop functions *)
5 (* and treat it as massless *)
6
7 ConsideredProcess = "2Fermion1Scalar";
8 FermionOrderExternal={2,1};
9 NeglectMasses={3};
10
11
12 ExternalFields= {DownQuark, bar[DownQuark], PseudoScalar};
13 CombinationGenerations = {{2,1,ALL},{3,1,ALL},{3,2,ALL}};
14
15
16 AllOperators={{OAh2qSL,Op[7]},
17               {OAh2qSR,Op[6]}},
18 };
19
20 OutputFile = "A2q.m";
21
22 Filters = {};

```

B.2: 4-Fermion operators

All operators listed below carry four indices and have dimension $3 \times 3 \times 3 \times 3$. In addition, the user can access the different contributions of all operators from tree-level diagrams, as well as penguin and box diagrams. The name conventions are as follows: for each operator `op` the additional parameter exist

- **TSop**: tree-level contributions with scalar propagator
- **TVop**: tree-level contributions with scalar propagator
- **PSop**: sum of penguin and self-energy contributions with scalar propagator
- **PVop**: sum of penguin and self-energy contributions with scalar propagator
- **Bop**: box contributions.

We will denote the 4-fermion operators involving two leptons and two down-type quarks depending on whether they lead to LFV or to QFV processes: $\ell\ell dd$ for LFV and $dd\ell\ell$ for QFV.

$$(\bar{d}\Gamma d)(\bar{\ell}\Gamma'\ell) \text{ and } (\bar{d}\Gamma d)(\bar{\nu}\Gamma'\nu)$$

Variable	Operator	Name	Variable	Operator	Name
OddllSLL	$(\bar{d}P_L d)(\bar{\ell}P_L \ell)$	E_{LL}^S			
OddllSRR	$(\bar{d}P_R d)(\bar{\ell}P_R \ell)$	E_{RR}^S			
OddllSLR	$(\bar{d}P_L d)(\bar{\ell}P_R \ell)$	E_{LR}^S			
OddllSRL	$(\bar{d}P_R d)(\bar{\ell}P_L \ell)$	E_{RL}^S			
OddllVLL	$(\bar{d}\gamma_\mu P_L d)(\bar{\ell}\gamma^\mu P_L \ell)$	E_{LL}^V	OddvvVLL	$(\bar{d}\gamma_\mu P_L d)(\bar{\nu}\gamma^\mu P_R \nu)$	F_{LL}^V
OddllVRR	$(\bar{d}\gamma_\mu P_R d)(\bar{\ell}\gamma^\mu P_R \ell)$	E_{RR}^V	OddvvVRR	$(\bar{d}\gamma_\mu P_R d)(\bar{\nu}\gamma^\mu P_R \nu)$	F_{RR}^V
OddllVLR	$(\bar{d}\gamma_\mu P_L d)(\bar{\ell}\gamma^\mu P_R \ell)$	E_{LR}^V	OddvvVLR	$(\bar{d}\gamma_\mu P_L d)(\bar{\nu}\gamma^\mu P_R \nu)$	F_{LR}^V
OddllVRL	$(\bar{d}\gamma_\mu P_R d)(\bar{\ell}\gamma^\mu P_L \ell)$	E_{RL}^V	OddvvVRL	$(\bar{d}\gamma_\mu P_R d)(\bar{\nu}\gamma^\mu P_L \nu)$	F_{RL}^V
OddllTLL	$(\bar{d}\sigma_{\mu\nu} P_L d)(\bar{\ell}\sigma^{\mu\nu} P_L \ell)$	E_{LL}^T			
OddllTRR	$(\bar{d}\sigma_{\mu\nu} P_R d)(\bar{\ell}\sigma^{\mu\nu} P_R \ell)$	E_{RR}^T			
OddllTLR	$(\bar{d}\sigma_{\mu\nu} P_L d)(\bar{\ell}\sigma^{\mu\nu} P_R \ell)$	E_{LR}^T			
OddllTRL	$(\bar{d}\sigma_{\mu\nu} P_R d)(\bar{\ell}\sigma^{\mu\nu} P_L \ell)$	E_{RL}^T			

These operators are derived by **PreSARAH** with the following input files

Listing 11 2d2L.m

```

1 NameProcess="2d2L ";
2
3 ConsideredProcess = "4Fermion";
4 FermionOrderExternal={2,1,4,3};
5 NeglectMasses={1,2,3,4};
6
7
8 ExternalFields= {DownQuark, bar [DownQuark] , ChargedLepton , bar [ChargedLepton] };
9
10 CombinationGenerations = {{3,1,1,1}, {3,1,2,2}, {3,1,3,3},
11                             {3,2,1,1}, {3,2,2,2}, {3,2,3,3}};
12
13
14 AllOperators={{OddllSLL,Op[7].Op[7]},
15               {OddllSRR,Op[6].Op[6]},
16               {OddllSRL,Op[6].Op[7]},
17               {OddllSLR,Op[7].Op[6]},
18
19               {OddllVRR,Op[7,Lor[1]].Op[7,Lor[1]]},
20               {OddllVLL,Op[6,Lor[1]].Op[6,Lor[1]]},
21               {OddllVRL,Op[7,Lor[1]].Op[6,Lor[1]]},
22               {OddllVLR,Op[6,Lor[1]].Op[7,Lor[1]]},
23
24               {OddllTLL,Op[-7,Lor[1],Lor[2]].Op[-7,Lor[1],Lor[2]]},
25               {OddllTLR,Op[-7,Lor[1],Lor[2]].Op[-6,Lor[1],Lor[2]]},
26               {OddllTRL,Op[-6,Lor[1],Lor[2]].Op[-7,Lor[1],Lor[2]]},
27               {OddllTRR,Op[-6,Lor[1],Lor[2]].Op[-6,Lor[1],Lor[2]]}
28 };

```

Listing 12 2d2nu.m

```

1 NameProcess="2d2nu ";
2
3 ConsideredProcess = "4Fermion";
4 FermionOrderExternal={2,1,4,3};
5 NeglectMasses={1,2,3,4};
6
7 ExternalFields= {DownQuark, bar [DownQuark] , Neutrino , bar [Neutrino] };
8
9 CombinationGenerations = Flatten[Table[{2,1, neutrino1, neutrino2},
10   {3,1, neutrino1, neutrino2},{3,2, neutrino1, neutrino2}},
11   {neutrino1,1,3},{neutrino2,1,3}],2];
12
13
14 AllOperators={{OddvvVRR,Op[7,Lor[1]].Op[7,Lor[1]]},
15               {OddvvVLL,Op[6,Lor[1]].Op[6,Lor[1]]},
16               {OddvvVRL,Op[7,Lor[1]].Op[6,Lor[1]]},
17               {OddvvVLR,Op[6,Lor[1]].Op[7,Lor[1]]}
18 };

```

$$(\bar{\ell}\Gamma\ell)(\bar{d}\Gamma'd) \text{ and } (\bar{\ell}\Gamma\ell)(\bar{u}\Gamma'u)$$

Variable	Operator	Name	Variable	Operator	Name
OllddSLL	$(\bar{\ell}P_L\ell)(\bar{d}P_Ld)$	B_{LL}^S	OlluuSLL	$(\bar{\ell}P_L\ell)(\bar{u}P_Lu)$	C_{LL}^S
OllddSRR	$(\bar{\ell}P_R\ell)(\bar{d}P_Rd)$	B_{RR}^S	OlluuSRR	$(\bar{\ell}P_R\ell)(\bar{u}P_Ru)$	C_{RR}^S
OllddSRL	$(\bar{\ell}P_R\ell)(\bar{d}P_Ld)$	B_{RL}^S	OlluuSRL	$(\bar{\ell}P_R\ell)(\bar{u}P_Lu)$	C_{RL}^S
OllddSLR	$(\bar{\ell}P_L\ell)(\bar{d}P_Rd)$	B_{LR}^S	OlluuSLR	$(\bar{\ell}P_L\ell)(\bar{u}P_Ru)$	C_{LR}^S
OllddVLL	$(\bar{\ell}\gamma_\mu P_L\ell)(\bar{d}\gamma^\mu P_Ld)$	B_{LL}^V	OlluuVLL	$(\bar{\ell}\gamma_\mu P_L\ell)(\bar{u}\gamma^\mu P_Lu)$	C_{LL}^V
OllddVRR	$(\bar{\ell}\gamma_\mu P_R\ell)(\bar{d}\gamma^\mu P_Rd)$	B_{RR}^V	OlluuVRR	$(\bar{\ell}\gamma_\mu P_R\ell)(\bar{u}\gamma^\mu P_Ru)$	C_{RR}^V
OllddVLR	$(\bar{\ell}\gamma_\mu P_L\ell)(\bar{d}\gamma^\mu P_Rd)$	B_{LR}^V	OlluuVLR	$(\bar{\ell}\gamma_\mu P_L\ell)(\bar{u}\gamma^\mu P_Ru)$	C_{LR}^V
OllddVRL	$(\bar{\ell}\gamma_\mu P_R\ell)(\bar{d}\gamma^\mu P_Ld)$	B_{RL}^V	OlluuVRL	$(\bar{\ell}\gamma_\mu P_R\ell)(\bar{u}\gamma^\mu P_Lu)$	C_{RL}^V
OllddTLL	$(\bar{\ell}\sigma_{\mu\nu}P_L\ell)(\bar{d}\sigma^{\mu\nu}P_Ld)$	B_{LL}^T	OlluuTLL	$(\bar{\ell}\sigma_{\mu\nu}P_L\ell)(\bar{u}\sigma^{\mu\nu}P_Lu)$	C_{LL}^T
OllddTRR	$(\bar{\ell}\sigma_{\mu\nu}P_R\ell)(\bar{d}\sigma^{\mu\nu}P_Rd)$	B_{RR}^T	OlluuTRR	$(\bar{\ell}\sigma_{\mu\nu}P_R\ell)(\bar{u}\sigma^{\mu\nu}P_Ru)$	C_{RR}^T
OllddTLR	$(\bar{\ell}\sigma_{\mu\nu}P_L\ell)(\bar{d}\sigma^{\mu\nu}P_Rd)$	B_{LR}^T	OlluuTLR	$(\bar{\ell}\sigma_{\mu\nu}P_L\ell)(\bar{u}\sigma^{\mu\nu}P_Ru)$	C_{LR}^T
OllddTRL	$(\bar{\ell}\sigma_{\mu\nu}P_R\ell)(\bar{d}\sigma^{\mu\nu}P_Ld)$	B_{RL}^T	OlluuTRL	$(\bar{\ell}\sigma_{\mu\nu}P_R\ell)(\bar{u}\sigma^{\mu\nu}P_Lu)$	C_{RL}^T

Listing 13 2L2d.m

```

1 NameProcess="2L2d ";
2
3 ConsideredProcess = "4Fermion ";
4 FermionOrderExternal={2,1,4,3};
5 NeglectMasses={1,2,3,4};
6
7
8 ExternalFields= {ChargedLepton,bar[ChargedLepton],DownQuark,bar[DownQuark]};
9 CombinationGenerations = {{2,1,1,1},{3,1,1,1},{3,2,1,1},
10                             {2,1,2,2},{3,1,2,2},{3,2,2,2}};
11
12
13 AllOperators={{OllddSLL,Op[7].Op[7]},
14               {OllddSRR,Op[6].Op[6]},
15               {OllddSRL,Op[6].Op[7]},
16               {OllddSLR,Op[7].Op[6]},
17
18               {OllddVRR,Op[7,Lor[1]].Op[7,Lor[1]]},
19               {OllddVLL,Op[6,Lor[1]].Op[6,Lor[1]]},
20               {OllddVRL,Op[7,Lor[1]].Op[6,Lor[1]]},
21               {OllddVLR,Op[6,Lor[1]].Op[7,Lor[1]]},
22
23               {OllddTLL,Op[-7,Lor[1],Lor[2]].Op[-7,Lor[1],Lor[2]]},
24               {OllddTLR,Op[-7,Lor[1],Lor[2]].Op[-6,Lor[1],Lor[2]]},
25               {OllddTRL,Op[-6,Lor[1],Lor[2]].Op[-7,Lor[1],Lor[2]]},
26               {OllddTRR,Op[-6,Lor[1],Lor[2]].Op[-6,Lor[1],Lor[2]]}
27 };

```

Listing 14 2L2u.m

```

1 NameProcess="2L2u ";
2
3 ConsideredProcess = "4Fermion ";
4 FermionOrderExternal={2,1,4,3};
5 NeglectMasses={1,2,3,4};
6
7
8 ExternalFields= {ChargedLepton,bar[ChargedLepton],UpQuark,bar[UpQuark]};
9 CombinationGenerations = {{2,1,1,1},{3,1,1,1},{3,2,1,1}};
10
11
12
13 AllOperators={{OlluuSLL,Op[7].Op[7]},
14               {OlluuSRR,Op[6].Op[6]},
15               {OlluuSRL,Op[6].Op[7]},
16               {OlluuSLR,Op[7].Op[6]},

```

```

17      {OlluuVRR, Op[7, Lor[1]] . Op[7, Lor[1]]},
18      {OlluuVLL, Op[6, Lor[1]] . Op[6, Lor[1]]},
19      {OlluuVRL, Op[7, Lor[1]] . Op[6, Lor[1]]},
20      {OlluuVLR, Op[6, Lor[1]] . Op[7, Lor[1]]},
21
22      {OlluuTLL, Op[-7, Lor[1], Lor[2]] . Op[-7, Lor[1], Lor[2]]},
23      {OlluuTLR, Op[-7, Lor[1], Lor[2]] . Op[-6, Lor[1], Lor[2]]},
24      {OlluuTRL, Op[-6, Lor[1], Lor[2]] . Op[-7, Lor[1], Lor[2]]},
25      {OlluuTRR, Op[-6, Lor[1], Lor[2]] . Op[-6, Lor[1], Lor[2]]}
26  };
27

```

$(\bar{d}\Gamma d)(\bar{d}\Gamma' d)$ and $(\bar{\ell}\Gamma\ell)(\bar{\ell}\Gamma'\ell)$

Variable	Operator	Name	Variable	Operator	Name
O4dSLL	$(\bar{d}P_L d)(\bar{d}P_L d)$	D_{LL}^S	O4lSLL	$(\bar{\ell}P_L \ell)(\bar{\ell}P_L \ell)$	A_{LL}^S
O4dSRR	$(\bar{d}P_R d)(\bar{d}P_R d)$	D_{RR}^S	O4lSRR	$(\bar{\ell}P_R \ell)(\bar{\ell}P_R \ell)$	A_{RR}^S
O4dSLR	$(\bar{d}P_L d)(\bar{d}P_R d)$	D_{LR}^S	O4lSLR	$(\bar{\ell}P_L \ell)(\bar{\ell}P_R \ell)$	A_{LR}^S
O4dSRL	$(\bar{d}P_R d)(\bar{d}P_L d)$	D_{RL}^S	O4lSRL	$(\bar{\ell}P_R \ell)(\bar{\ell}P_L \ell)$	A_{RL}^S
O4dVLL	$(\bar{d}\gamma_\mu P_L d)(\bar{d}\gamma^\mu P_L d)$	D_{LL}^V	O4lVLL	$(\bar{\ell}\gamma_\mu P_L \ell)(\bar{\ell}\gamma^\mu P_L \ell)$	A_{LL}^V
O4dVRR	$(\bar{d}\gamma_\mu P_R d)(\bar{d}\gamma^\mu P_R d)$	D_{RR}^V	O4lVRR	$(\bar{\ell}\gamma_\mu P_R \ell)(\bar{\ell}\gamma^\mu P_R \ell)$	A_{RR}^V
O4dVLR	$(\bar{d}\gamma_\mu P_L d)(\bar{d}\gamma^\mu P_R d)$	D_{LR}^V	O4lVLR	$(\bar{\ell}\gamma_\mu P_L \ell)(\bar{\ell}\gamma^\mu P_R \ell)$	A_{LR}^V
O4dVRL	$(\bar{d}\gamma_\mu P_R d)(\bar{d}\gamma^\mu P_L d)$	D_{RL}^V	O4lVRL	$(\bar{\ell}\gamma_\mu P_R \ell)(\bar{\ell}\gamma^\mu P_L \ell)$	A_{RL}^V
O4dTLL	$(\bar{d}\sigma_{\mu\nu} P_L d)(\bar{d}\sigma^{\mu\nu} P_L d)$	D_{LL}^T	O4lTLL	$(\bar{\ell}\sigma_{\mu\nu} P_L \ell)(\bar{\ell}\sigma^{\mu\nu} P_L \ell)$	A_{LL}^T
O4dTRR	$(\bar{d}\sigma_{\mu\nu} P_R d)(\bar{d}\sigma^{\mu\nu} P_R d)$	D_{RR}^T	O4lTRR	$(\bar{\ell}\sigma_{\mu\nu} P_R \ell)(\bar{\ell}\sigma^{\mu\nu} P_R \ell)$	A_{RR}^T
O4dTLR	$(\bar{d}\sigma_{\mu\nu} P_L d)(\bar{d}\sigma^{\mu\nu} P_R d)$	D_{LR}^T	O4lTLR	$(\bar{\ell}\sigma_{\mu\nu} P_L \ell)(\bar{\ell}\sigma^{\mu\nu} P_R \ell)$	A_{LR}^T
O4dTRL	$(\bar{d}\sigma_{\mu\nu} P_R d)(\bar{d}\sigma^{\mu\nu} P_L d)$	D_{RL}^T	O4lTRL	$(\bar{\ell}\sigma_{\mu\nu} P_R \ell)(\bar{\ell}\sigma^{\mu\nu} P_L \ell)$	A_{RL}^T

Listing 15 4d.m

```

1  NameProcess="4d ";
2
3  ConsideredProcess = "4Fermion ";
4  FermionOrderExternal={2,1,4,3};
5  NeglectMasses={1,2,3,4};
6
7
8  ExternalFields= {DownQuark, bar[DownQuark], DownQuark, bar[DownQuark]};
9
10 ColorFlow = ColorDelta[1,2] ColorDelta[3,4];
11
12 CombinationGenerations = {{3,1,3,1},{3,2,3,2},{2,1,2,1}};
13
14
15 AllOperators={{O4dSLL, Op[7] . Op[7]},
16               {O4dSRR, Op[6] . Op[6]},
17               {O4dSRL, Op[6] . Op[7]},
18               {O4dSLR, Op[7] . Op[6]},
19
20               {O4dVRR, Op[7, Lor[1]] . Op[7, Lor[1]]},
21               {O4dVLL, Op[6, Lor[1]] . Op[6, Lor[1]]},
22               {O4dVRL, Op[7, Lor[1]] . Op[6, Lor[1]]},
23               {O4dVLR, Op[6, Lor[1]] . Op[7, Lor[1]]},
24
25               {O4dTLL, Op[-7, Lor[1], Lor[2]] . Op[-7, Lor[1], Lor[2]]},
26               {O4dTLR, Op[-7, Lor[1], Lor[2]] . Op[-6, Lor[1], Lor[2]]},
27               {O4dTRL, Op[-6, Lor[1], Lor[2]] . Op[-7, Lor[1], Lor[2]]},
28               {O4dTRR, Op[-6, Lor[1], Lor[2]] . Op[-6, Lor[1], Lor[2]]}
29  };
30
31 Filters = {NoPenguins};

```

Listing 16 4L.m

```

1 NameProcess="4L";
2
3 ConsideredProcess = "4Fermion";
4 FermionOrderExternal={2,1,4,3};
5 NeglectMasses={1,2,3,4};
6
7 ExternalFields= {ChargedLepton, bar [ChargedLepton], ChargedLepton, bar [ChargedLepton]};
8 CombinationGenerations = {{2,1,1,1},{3,1,1,1},{3,2,2,2}};
9
10
11 AllOperators={{O4lSLL, Op[7].Op[7]},
12               {O4lSRR, Op[6].Op[6]},
13               {O4lSRL, Op[6].Op[7]},
14               {O4lSLR, Op[7].Op[6]},
15
16               {O4lVRR, Op[7, Lor[1]].Op[7, Lor[1]]},
17               {O4lVLL, Op[6, Lor[1]].Op[6, Lor[1]]},
18               {O4lVRL, Op[7, Lor[1]].Op[6, Lor[1]]},
19               {O4lVLR, Op[6, Lor[1]].Op[7, Lor[1]]},
20
21               {O4lTLL, Op[-7, Lor[1], Lor[2]].Op[-7, Lor[1], Lor[2]]},
22               {O4lTLR, Op[-7, Lor[1], Lor[2]].Op[-6, Lor[1], Lor[2]]},
23               {O4lTRL, Op[-6, Lor[1], Lor[2]].Op[-7, Lor[1], Lor[2]]},
24               {O4lTRR, Op[-6, Lor[1], Lor[2]].Op[-6, Lor[1], Lor[2]]}
25 };
26
27 Filters = {NoCrossedDiagrams};

```

$$(\bar{d}\Gamma u)(\bar{\ell}\Gamma'\nu)$$

Variable	Operator	Name	Variable	Operator	Name
OdulvVLL	$(\bar{d}\gamma_\mu P_L u)(\bar{\ell}\gamma^\mu P_L \nu)$	G_{LL}^V	OdulvSLL	$(\bar{d}P_L u)(\bar{\ell}P_L \nu)$	G_{LL}^S
OdulvVRR	$(\bar{d}\gamma_\mu P_R u)(\bar{\ell}\gamma^\mu P_R \nu)$	G_{RR}^V	OdulvSRR	$(\bar{d}P_R u)(\bar{\ell}P_R \nu)$	G_{RR}^S
OdulvVLR	$(\bar{d}\gamma_\mu P_L u)(\bar{\ell}\gamma^\mu P_R \nu)$	G_{LR}^V	OdulvSLR	$(\bar{d}P_L u)(\bar{\ell}P_R \nu)$	G_{LR}^S
OdulvVRL	$(\bar{d}\gamma_\mu P_R u)(\bar{\ell}\gamma^\mu P_L \nu)$	G_{RL}^V	OdulvSRL	$(\bar{d}P_R u)(\bar{\ell}P_L \nu)$	G_{RL}^S

Listing 17 du_lv.m

```

1 NameProcess="dulv";
2
3 ConsideredProcess = "4Fermion";
4 FermionOrderExternal={2,1,3,4};
5 NeglectMasses={1,2,3,4};
6
7
8 ExternalFields= {DownQuark, bar [UpQuark], Neutrino, bar [ChargedLepton]};
9
10 CombinationGenerations =
11   Flatten [Table[{{3,1,i,j},{3,2,i,j},{2,2,i,j},{2,1,i,j},{i,1,3},{j,1,3}},2];
12
13 Clear [i,j];
14
15
16 AllOperators={{OdulvSLL, Op[7].Op[7]},
17               {OdulvSRR, Op[6].Op[6]},
18               {OdulvSRL, Op[6].Op[7]},
19               {OdulvSLR, Op[7].Op[6]},
20
21               {OdulvVRR, Op[7, Lor[1]].Op[7, Lor[1]]},
22               {OdulvVLL, Op[6, Lor[1]].Op[6, Lor[1]]},
23               {OdulvVRL, Op[7, Lor[1]].Op[6, Lor[1]]},
24               {OdulvVLR, Op[6, Lor[1]].Op[7, Lor[1]]}
25 };

```

```

26 Filters = {NoBoxes, NoPenguins};
27

```

C: Application: Flavor observables implemented in SARAH

C.1: Lepton flavor observables

Lepton flavor violation in the SM or MSSM without neutrino masses vanishes exactly. Even adding Dirac neutrino masses to the SM predicts LFV rates which are far beyond the experimental reach. However, many extensions of the SM can introduce new sources for LFV of a size which is testable nowadays. The best-known examples are SUSY and non-SUSY models with high- or low-scale seesaw mechanism, models with vector-like leptons and SUSY models with R -parity violation, see for instance Refs. [32, 42, 58–89].

We discuss in the following the implementation of the most important LFV observables in **SARAH** and **SPheno** using the previously defined operators which are calculated by **SPheno**.

C.1.1: $\ell_\alpha \rightarrow \ell_\beta \gamma$

The decay width is given by [42]

$$\Gamma(\ell_\alpha \rightarrow \ell_\beta \gamma) = \frac{\alpha m_{\ell_\alpha}^5}{4} \left(|K_2^L|^2 + |K_2^R|^2 \right), \quad (\text{C.18})$$

where α is the fine structure constant and the dipole Wilson coefficients $K_2^{L,R}$ are defined in Eq.(A.3).

Listing 18 LLgGamma.m

```

1 NameProcess = "LLpGamma";
2 NameObservables = {{muEgamma, 701, "BR(mu->e gamma)"},
3                   {tauEgamma, 702, "BR(tau->e gamma)"},
4                   {tauMuGamma, 703, "BR(tau->mu gamma)"};
5
6 NeededOperators = {K2L, K2R};
7
8 Body = "LLpGamma.f90 ";

```

Listing 19 LLgGamma.f90

```

1 Real(dp) :: width
2 Integer :: i1, gt1, gt2
3
4 ! -----
5 ! l -> l' gamma
6 ! Observable implemented by W. Porod, F. Staub and A. Vicente
7 ! Based on J. Hisano et al, PRD 53 (1996) 2442 [hep-ph/9510309]
8 ! -----
9
10 Do i1=1,3
11
12   If (i1.eq.1) Then           ! mu -> e gamma
13     gt1 = 2
14     gt2 = 1
15   ElseIf (i1.eq.2) Then      ! tau -> e gamma
16     gt1 = 3
17     gt2 = 1
18   Else                       ! tau -> mu gamma
19     gt1 = 3
20     gt2 = 2
21   End if
22
23   width=0.25_dp*mf_1(gt1)**5*(Abs(K2L(gt1,gt2))**2 &
24     & +Abs(K2R(gt1,gt2))**2)*Alpha

```



```

25 | If (i1.eq.1) Then
26 |   muEgamma = width/(width+GammaMu)
27 | Elseif (i1.eq.2) Then
28 |   tauEgamma = width/(width+GammaTau)
29 | Else
30 |   tauMuGamma = width/(width+GammaTau)
31 | End if
32 | End do
33 |
34 | End do

```

C.1.2: $\ell_\alpha \rightarrow 3\ell_\beta$

The decay width is given by

$$\begin{aligned}
\Gamma(\ell_\alpha \rightarrow 3\ell_\beta) = & \frac{m_{\ell_\alpha}^5}{512\pi^3} \left[e^4 \left(|K_2^L|^2 + |K_2^R|^2 \right) \left(\frac{16}{3} \log \frac{m_{\ell_\alpha}}{m_{\ell_\beta}} - \frac{22}{3} \right) \right. \\
& + \frac{1}{24} \left(|A_{LL}^S|^2 + |A_{RR}^S|^2 \right) + \frac{1}{12} \left(|A_{LR}^S|^2 + |A_{RL}^S|^2 \right) \\
& + \frac{2}{3} \left(|\hat{A}_{LL}^V|^2 + |\hat{A}_{RR}^V|^2 \right) + \frac{1}{3} \left(|\hat{A}_{LR}^V|^2 + |\hat{A}_{RL}^V|^2 \right) + 6 \left(|A_{LL}^T|^2 + |A_{RR}^T|^2 \right) \\
& + \frac{e^2}{3} \left(K_2^L A_{RL}^{S*} + K_2^R A_{LR}^{S*} + c.c. \right) - \frac{2e^2}{3} \left(K_2^L \hat{A}_{RL}^{V*} + K_2^R \hat{A}_{LR}^{V*} + c.c. \right) \\
& - \frac{4e^2}{3} \left(K_2^L \hat{A}_{RR}^{V*} + K_2^R \hat{A}_{LL}^{V*} + c.c. \right) \\
& \left. - \frac{1}{2} \left(A_{LL}^S A_{LL}^{T*} + A_{RR}^S A_{RR}^{T*} + c.c. \right) - \frac{1}{6} \left(A_{LR}^S \hat{A}_{LR}^{V*} + A_{RL}^S \hat{A}_{RL}^{V*} + c.c. \right) \right].
\end{aligned} \tag{C.19}$$

Here we have defined

$$\hat{A}_{XY}^V = A_{XY}^V + e^2 K_1^X \quad (X, Y = L, R). \tag{C.20}$$

The mass of the leptons in the final state has been neglected in this formula, with the exception of the dipole terms $K_2^{L,R}$, where an infrared divergence would otherwise occur due to the massless photon propagator. Eq.(C.19) is in agreement with [58], but also includes the coefficients A_{LR}^S and A_{RL}^S .

Listing 20 Lto3Lp.m

```

1 | NameProcess = "Lto3Lp ";
2 | NameObservables = { {BRmuTo3e, 901, "BR(mu->3e)"},
3 |                     {BRtauTo3e, 902, "BR(tau->3e)"},
4 |                     {BRtauTo3mu, 903, "BR(tau->3mu)"}
5 |                     };
6 |
7 | ExternalStates = {Electron};
8 | NeededOperators = {K1L, K1R, K2L, K2R,
9 |                   O4ISLL, O4ISRR, O4ISRL, O4ISLR,
10 |                  O4IVRR, O4IVLL, O4IVRL, O4IVLR,
11 |                  O4ITLL, O4ITRR };
12 |
13 | Body = "Lto3Lp.f90 ";

```

Listing 21 Lto3Lp.f90

```

1 | Complex(dp) :: cK1L, cK1R, cK2L, cK2R
2 | Complex(dp) :: CSL, CSRR, CSLR, CSRL, CVLL, &
3 |               & CVRR, CVLR, CVRL, CTLL, CTRR
4 | Real(dp) :: BRdipole, BRscalar, BRvector, BRtensor
5 | Real(dp) :: BRmix1, BRmix2, BRmix3, BRmix4, GammaLFV
6 | Real(dp) :: e2, e4
7 | Integer :: i1, gt1, gt2, gt3, gt4
8 |

```

```

9  ! -----
10 ! l -> 3 l'
11 ! Observable implemented by W. Porod, F. Staub and A. Vicente
12 ! -----
13
14 e2 = (4._dp*Pi*Alpha_MZ)
15 e4 = e2**2
16
17 Do i1=1,3
18
19   If (i1.eq.1) Then
20     gt1 = 2
21     gt2 = 1
22   ElseIf (i1.eq.2) Then
23     gt1 = 3
24     gt2 = 1
25   Else
26     gt1 = 3
27     gt2 = 2
28   End if
29   gt3 = gt2
30   gt4 = gt2
31
32   cK1L = K1L(gt1,gt2)
33   cK1R = K1R(gt1,gt2)
34
35   cK2L = K2L(gt1,gt2)
36   cK2R = K2R(gt1,gt2)
37
38   CSLL = O4ISLL(gt1,gt2,gt3,gt4)
39   CSRR = O4ISRR(gt1,gt2,gt3,gt4)
40   CSLR = O4ISLR(gt1,gt2,gt3,gt4)
41   CSRL = O4ISRL(gt1,gt2,gt3,gt4)
42
43   CVLL = O4IVLL(gt1,gt2,gt3,gt4)
44   CVRR = O4IVRR(gt1,gt2,gt3,gt4)
45   CVLR = O4IVLR(gt1,gt2,gt3,gt4)
46   CVRL = O4IVRL(gt1,gt2,gt3,gt4)
47
48   CVLL = CVLL + e2*cK1L
49   CVRR = CVRR + e2*cK1R
50   CVLR = CVLR + e2*cK1L
51   CVRL = CVRL + e2*cK1R
52
53   CTLL = O4ITLL(gt1,gt2,gt3,gt4)
54   CTRR = O4ITRR(gt1,gt2,gt3,gt4)
55
56   ! Photonic dipole contributions
57   BRdipole = (Abs(cK2L)**2+Abs(cK2R)**2)&
58   &*(16._dp*Log(mf_l(gt1)/mf_l(gt2))-22._dp)/3._dp
59
60   ! Scalar contributions
61   BRscalar = (Abs(CSLL)**2+Abs(CSRR)**2)/24._dp&
62   &+(Abs(CSLR)**2+Abs(CSRL)**2)/12._dp
63
64   ! Vector contributions
65   BRvector = 2._dp*(Abs(CVLL)**2+Abs(CVRR)**2)/3._dp&
66   &+(Abs(CVLR)**2+Abs(CVRL)**2)/3._dp
67
68   ! Tensor contributions
69   BRtensor = 6._dp*(Abs(CTLL)**2+Abs(CTRR)**2)
70
71   ! Mix: dipole x scalar
72   BRmix1 = 2._dp/3._dp*Real(cK2L*Conjg(CSRL) + cK2R*Conjg(CSLR),dp)
73
74   ! Mix: dipole x vector
75   BRmix2 = -4._dp/3._dp*Real(cK2L*Conjg(CVRL) + cK2R*Conjg(CVLR),dp) &
76   &-8._dp/3._dp*Real(cK2L*Conjg(CVRR) + cK2R*Conjg(CVLL),dp)
77

```

```

78 ! Mix: scalar x vector
79 BRmix3 = -1._dp/3._dp*Real(CSLR*Conjg(CVLR) + CSRL*Conjg(CVRL),dp)
80
81 ! Mix: scalar x tensor
82 BRmix4 = -1._dp*Real(CSLL*Conjg(CTLL) + CSRR*Conjg(CTRR),dp)
83
84 GammaLFV = oo512pi3*mf_l(gt1)**5* &
85   & (e4*BRdipole + BRscalar + BRvector + BRtensor &
86   & + e2*BRmix1 + e2*BRmix2 + BRmix3 + BRmix4)
87
88 !
89 !taking alpha(Q=0) instead of alpha(m_Z) as this contains most of the
90 !running of the Wilson coefficients
91 !
92
93 If (i1.Eq.1) Then
94   BRmuTo3e=GammaLFV/GammaMu
95 Else If (i1.Eq.2) Then
96   BRtauTo3e=GammaLFV/GammaTau
97 Else
98   BRtauTo3mu=GammaLFV/GammaTau
99 End If
100 End do

```

C.1.3: Coherent $\mu - e$ conversion in nuclei

The conversion rate, relative to the the muon capture rate, can be expressed as [90,91]

$$\begin{aligned}
\text{CR}(\mu - e, \text{Nucleus}) &= \frac{p_e E_e m_\mu^3 G_F^2 \alpha^3 Z_{\text{eff}}^4 F_p^2}{8 \pi^2 Z} \\
&\times \left\{ \left| (Z + N) \left(g_{LV}^{(0)} + g_{LS}^{(0)} \right) + (Z - N) \left(g_{LV}^{(1)} + g_{LS}^{(1)} \right) \right|^2 + \right. \\
&\quad \left. \left| (Z + N) \left(g_{RV}^{(0)} + g_{RS}^{(0)} \right) + (Z - N) \left(g_{RV}^{(1)} + g_{RS}^{(1)} \right) \right|^2 \right\} \frac{1}{\Gamma_{\text{capt}}}. \quad (\text{C.21})
\end{aligned}$$

Z and N are the number of protons and neutrons in the nucleus and Z_{eff} is the effective atomic charge [92]. Similarly, G_F is the Fermi constant, F_p is the nuclear matrix element and Γ_{capt} represents the total muon capture rate. α is the fine structure constant, p_e and E_e ($\simeq m_\mu$ in the numerical evaluation) are the momentum and energy of the electron and m_μ is the muon mass. In the above, $g_{XK}^{(0)}$ and $g_{XK}^{(1)}$ (with $X = L, R$ and $K = S, V$) can be written in terms of effective couplings at the quark level as

$$\begin{aligned}
g_{XK}^{(0)} &= \frac{1}{2} \sum_{q=u,d,s} \left(g_{XK(q)} G_K^{(q,p)} + g_{XK(q)} G_K^{(q,n)} \right), \\
g_{XK}^{(1)} &= \frac{1}{2} \sum_{q=u,d,s} \left(g_{XK(q)} G_K^{(q,p)} - g_{XK(q)} G_K^{(q,n)} \right). \quad (\text{C.22})
\end{aligned}$$

For coherent $\mu - e$ conversion in nuclei, only scalar (S) and vector (V) couplings contribute. Furthermore, sizable contributions are expected only from the u, d, s quark flavors. The numerical values of the relevant G_K factors are [90,93]

$$\begin{aligned}
G_V^{(u,p)} &= G_V^{(d,n)} = 2; & G_V^{(d,p)} &= G_V^{(u,n)} = 1; \\
G_S^{(u,p)} &= G_S^{(d,n)} = 5.1; & G_S^{(d,p)} &= G_S^{(u,n)} = 4.3; \\
G_S^{(s,p)} &= G_S^{(s,n)} = 2.5. \quad (\text{C.23})
\end{aligned}$$

Finally, the $g_{XK(q)}$ coefficients can be written in terms of the Wilson coefficients in Eqs.(A.3), (A.8) and (A.9) as

$$g_{LV(q)} = \frac{\sqrt{2}}{G_F} \left[e^2 Q_q \left(K_1^L - K_2^R \right) - \frac{1}{2} \left(C_{\ell\ell qq}^{VLL} + C_{\ell\ell qq}^{VLR} \right) \right] \quad (C.24)$$

$$g_{RV(q)} = g_{LV(q)}|_{L \rightarrow R} \quad (C.25)$$

$$g_{LS(q)} = -\frac{\sqrt{2}}{G_F} \frac{1}{2} \left(C_{\ell\ell qq}^{SLL} + C_{\ell\ell qq}^{SLR} \right) \quad (C.26)$$

$$g_{RS(q)} = g_{LS(q)}|_{L \rightarrow R} . \quad (C.27)$$

Here Q_q is the quark electric charge ($Q_d = -1/3$, $Q_u = 2/3$) and $C_{\ell\ell qq}^{IXK} = B_{XY}^K \left(C_{XY}^K \right)$ for d-quarks (u-quarks), with $X = L, R$ and $K = S, V$.

Listing 22 MuEconversion.m

```

1 NameProcess = "MuEconversion";
2 NameObservables = {{CRmuEAl, 800, "CR(mu-e, Al)"},
3                     {CRmuETi, 801, "CR(mu-e, Ti)"},
4                     {CRmuESr, 802, "CR(mu-e, Sr)"},
5                     {CRmuESb, 803, "CR(mu-e, Sb)"},
6                     {CRmuEAu, 804, "CR(mu-e, Au)"},
7                     {CRmuEPb, 805, "CR(mu-e, Pb)"}
8                     };
9
10 NeededOperators = {K1L, K1R, K2L, K2R,
11                    OllddSSL, OllddSRR, OllddSRL, OllddSLR, OllddVRR, OllddVLL,
12                    OllddVRL, OllddVLR, OllddTLL, OllddTLR, OllddTRL, OllddTRR,
13                    OlluuSSL, OlluuSRR, OlluuSRL, OlluuSLR, OlluuVRR, OlluuVLL,
14                    OlluuVRL, OlluuVLR, OlluuTLL, OlluuTLR, OlluuTRL, OlluuTRR
15                    };
16
17 Body = "MuEconversion.f90";

```

Listing 23 MuEconversion.f90

```

1 Complex(dp) :: gPLV(3), gPRV(3)
2 Complex(dp), Parameter :: mat0(3,3)=0._dp
3 Real(dp) :: Znuc, Nnuc, Zeff, Fp, GammaCapt, GSp(3), GSn(3), &
4           & Gvp(3), Gvn(3), e2
5 Complex(dp) :: Lcont, Rcont, gLS(3), gRS(3), gLV(3), gRV(3), g0LS, g0RS, &
6           & g0LV, g0RV, g1LS, g1RS, g1LV, g1RV
7 Integer :: i1, i2
8
9 ! -----
10 ! Coherent mu-e conversion in nuclei
11 ! Observable implemented by W. Porod, F. Staub and A. Vicente
12 ! Based on Y. Kuno, Y. Okada, Rev. Mod. Phys. 73 (2001) 151 [hep-ph/9909265]
13 ! and E. Arganda et al, JHEP 0710 (2007) 104 [arXiv:0707.2955]
14 ! -----
15
16 e2 = 4._dp*Pi*Alpha_MZ
17
18 ! 1: uu
19 ! 2: dd
20 ! 3: ss
21
22 ! vector couplings
23
24 gLV(1) = 0.5_dp*(OlluuVLL(2,1,1,1) + OlluuVLR(2,1,1,1))
25 gRV(1) = 0.5_dp*(OlluuVRL(2,1,1,1) + OlluuVRR(2,1,1,1))
26 gLV(2) = 0.5_dp*(OllddVLL(2,1,1,1) + OllddVLR(2,1,1,1))
27 gRV(2) = 0.5_dp*(OllddVRL(2,1,1,1) + OllddVRR(2,1,1,1))
28 gLV(3) = 0.5_dp*(OllddVLL(2,1,2,2) + OllddVLR(2,1,2,2))
29 gRV(3) = 0.5_dp*(OllddVRL(2,1,2,2) + OllddVRR(2,1,2,2))

```

```

30
31 gLV = -gLV*Sqrt(2._dp)/G_F
32 gRV = -gRV*Sqrt(2._dp)/G_F
33
34 gPLV(1) = (K1L(2,1)-K2R(2,1))*(2._dp/3._dp)
35 gPRV(1) = (K1R(2,1)-K2L(2,1))*(2._dp/3._dp)
36 gPLV(2) = (K1L(2,1)-K2R(2,1))*(-1._dp/3._dp)
37 gPRV(2) = (K1R(2,1)-K2L(2,1))*(-1._dp/3._dp)
38 gPLV(3) = (K1L(2,1)-K2R(2,1))*(-1._dp/3._dp)
39 gPRV(3) = (K1R(2,1)-K2L(2,1))*(-1._dp/3._dp)
40 gPLV = gPLV*Sqrt(2._dp)/G_F*e2
41 gPRV = gPRV*Sqrt(2._dp)/G_F*e2
42
43 gLV=gPLV+gLV
44 gRV=gPRV+gRV
45
46
47 ! scalar couplings
48
49 gLS(1) = 0.5_dp*(OlluuSLL(2,1,1,1)+OlluuSLR(2,1,1,1))
50 gRS(1) = 0.5_dp*(OlluuSRL(2,1,1,1)+OlluuSRR(2,1,1,1))
51 gLS(2) = 0.5_dp*(OllddSLL(2,1,1,1)+OllddSLR(2,1,1,1))
52 gRS(2) = 0.5_dp*(OllddSRL(2,1,1,1)+OllddSRR(2,1,1,1))
53 gLS(3) = 0.5_dp*(OllddSLL(2,1,2,2)+OllddSLR(2,1,2,2))
54 gRS(3) = 0.5_dp*(OllddSRL(2,1,2,2)+OllddSRR(2,1,2,2))
55
56 gLS = -gLS*Sqrt(2._dp)/G_F
57 gRS = -gRS*Sqrt(2._dp)/G_F
58
59
60 Do i1=1,6
61   If(i1.eq.1) Then
62     Znuc=13._dp
63     Nnuc=14._dp
64     Zeff=11.5_dp
65     Fp=0.64_dp
66     GammaCapt=4.64079e-19_dp
67   Else If(i1.eq.2) Then
68     Znuc=22._dp
69     Nnuc=26._dp
70     Zeff=17.6_dp
71     Fp=0.54_dp
72     GammaCapt=1.70422e-18_dp
73   Else If(i1.eq.3) Then
74     Znuc=38._dp
75     Nnuc=42._dp
76     Zeff=25.0_dp
77     Fp=0.39_dp
78     GammaCapt=4.61842e-18_dp
79   Else If(i1.eq.4) Then
80     Znuc=51._dp
81     Nnuc=70._dp
82     Zeff=29.0_dp
83     Fp=0.32_dp
84     GammaCapt=6.71711e-18_dp
85   Else If(i1.eq.5) Then
86     Znuc=79._dp
87     Nnuc=118._dp
88     Zeff=33.5_dp
89     Fp=0.16_dp
90     GammaCapt=8.59868e-18_dp
91   Else If(i1.eq.6) Then
92     Znuc=82._dp
93     Nnuc=125._dp
94     Zeff=34.0_dp
95     Fp=0.15_dp
96     GammaCapt=8.84868e-18_dp
97   End If
98

```

```

99  ! numerical values
100 ! based on Y. Kuno, Y. Okada, Rev. Mod. Phys. 73 (2001) 151 [hep-ph/9909265]
101 ! and T. S. Kosmas et al, PLB 511 (2001) 203 [hep-ph/0102101]
102 GSp=(/5.1,4.3,2.5/)
103 GSn=(/4.3,5.1,2.5/)
104 GVp=(/2.0,1.0,0.0/)
105 GVn=(/1.0,2.0,0.0/)
106
107 g0LS=0._dp
108 g0RS=0._dp
109 g0LV=0._dp
110 g0RV=0._dp
111 g1LS=0._dp
112 g1RS=0._dp
113 g1LV=0._dp
114 g1RV=0._dp
115 Do i2=1,3
116 g0LS=g0LS+0.5_dp*gLS(i2)*(GSp(i2)+GSn(i2))
117 g0RS=g0RS+0.5_dp*gRS(i2)*(GSp(i2)+GSn(i2))
118 g0LV=g0LV+0.5_dp*gLV(i2)*(GVp(i2)+GVn(i2))
119 g0RV=g0RV+0.5_dp*gRV(i2)*(GVp(i2)+GVn(i2))
120 g1LS=g1LS+0.5_dp*gLS(i2)*(GSp(i2)-GSn(i2))
121 g1RS=g1RS+0.5_dp*gRS(i2)*(GSp(i2)-GSn(i2))
122 g1LV=g1LV+0.5_dp*gLV(i2)*(GVp(i2)-GVn(i2))
123 g1RV=g1RV+0.5_dp*gRV(i2)*(GVp(i2)-GVn(i2))
124 End Do
125 Lcont=(Znuc+Nnuc)*(g0LV+g0LS)+(Znuc-Nnuc)*(g1LV-g1LS)
126 Rcont=(Znuc+Nnuc)*(g0RV+g0RS)+(Znuc-Nnuc)*(g1RV-g1RS)
127
128 ! Conversion rate
129 If (i1.eq.1) Then
130   CRMuEAl=oo8pi2*mf_l(2)**5*G_F**2*Alpha**3*Zeff**4*Fp**2/Znuc*&
131     & (Abs(Lcont)**2+Abs(Rcont)**2)/GammaCapt
132 Else if (i1.eq.2) Then
133   CRMuETi=oo8pi2*mf_l(2)**5*G_F**2*Alpha**3*Zeff**4*Fp**2/Znuc*&
134     & (Abs(Lcont)**2+Abs(Rcont)**2)/GammaCapt
135 Else if (i1.eq.3) Then
136   CRMuESr=oo8pi2*mf_l(2)**5*G_F**2*Alpha**3*Zeff**4*Fp**2/Znuc*&
137     & (Abs(Lcont)**2+Abs(Rcont)**2)/GammaCapt
138 Else if (i1.eq.4) Then
139   CRMuESb=oo8pi2*mf_l(2)**5*G_F**2*Alpha**3*Zeff**4*Fp**2/Znuc*&
140     & (Abs(Lcont)**2+Abs(Rcont)**2)/GammaCapt
141 Else if (i1.eq.5) Then
142   CRMuEAu=oo8pi2*mf_l(2)**5*G_F**2*Alpha**3*Zeff**4*Fp**2/Znuc*&
143     & (Abs(Lcont)**2+Abs(Rcont)**2)/GammaCapt
144 Else if (i1.eq.6) Then
145   CRMuEPb=oo8pi2*mf_l(2)**5*G_F**2*Alpha**3*Zeff**4*Fp**2/Znuc*&
146     & (Abs(Lcont)**2+Abs(Rcont)**2)/GammaCapt
147 End if
148 End do

```

C.1.4: $\tau \rightarrow P\ell$

Our analytical expressions for $\tau \rightarrow P\ell$, where $\ell = e, \mu$ and P is a pseudoscalar meson, generalize the results in [94]. The decay width is given by

$$\Gamma(\tau \rightarrow \ell P) = \frac{1}{4\pi} \frac{\lambda^{1/2}(m_\tau^2, m_\ell^2, m_P^2)}{m_\tau^2} \frac{1}{2} \sum_{i,f} |\mathcal{M}_{\tau\ell P}|^2, \quad (\text{C.28})$$

where the averaged squared amplitude can be written as

$$\frac{1}{2} \sum_{i,f} |\mathcal{M}_{\tau\ell P}|^2 = \frac{1}{4m_\tau} \sum_{I,J=S,V} \left[2m_\tau m_\ell \left(a_P^I a_P^{J*} - b_P^I b_P^{J*} \right) + (m_\tau^2 + m_\ell^2 - m_P^2) \left(a_P^I a_P^{J*} + b_P^I b_P^{J*} \right) \right]. \quad (\text{C.29})$$

The coefficients $a_P^{S,V}$ and $b_P^{S,V}$ can be expressed in terms of the Wilson coefficients in Eqs.(A.8) and (A.9) as

$$a_P^S = \frac{1}{2} f_\pi \sum_{X=L,R} \left[\frac{D_X^d(P)}{m_d} (B_{LX}^S + B_{RX}^S) + \frac{D_X^u(P)}{m_u} (C_{LX}^S + C_{RX}^S) \right] \quad (C.30)$$

$$b_P^S = \frac{1}{2} f_\pi \sum_{X=L,R} \left[\frac{D_X^d(P)}{m_d} (B_{RX}^S - B_{LX}^S) + \frac{D_X^u(P)}{m_u} (C_{RX}^S - C_{LX}^S) \right] \quad (C.31)$$

$$a_P^V = \frac{1}{4} f_\pi C(P) (m_\tau - m_\ell) \left[-B_{LL}^V + B_{LR}^V - B_{RL}^V + B_{RR}^V + C_{LL}^V - C_{LR}^V + C_{RL}^V - C_{RR}^V \right] \quad (C.32)$$

$$b_P^V = \frac{1}{4} f_\pi C(P) (m_\tau + m_\ell) \left[-B_{LL}^V + B_{LR}^V + B_{RL}^V - B_{RR}^V + C_{LL}^V - C_{LR}^V - C_{RL}^V + C_{RR}^V \right]. \quad (C.33)$$

In these expressions m_d and m_u are the down- and up-quark masses, respectively, f_π is the pion decay constant and the coefficients $C(P)$, $D_{L,R}^{d,u}(P)$ take different forms for each pseudoscalar meson P [94]. For $P = \pi$ one has

$$C(\pi) = 1 \quad (C.34)$$

$$D_L^d(\pi) = -\frac{m_\pi^2}{4} \quad (C.35)$$

$$D_L^u(\pi) = \frac{m_\pi^2}{4}, \quad (C.36)$$

for $P = \eta$

$$C(\eta) = \frac{1}{\sqrt{6}} (\sin \theta_\eta + \sqrt{2} \cos \theta_\eta) \quad (C.37)$$

$$D_L^d(\eta) = \frac{1}{4\sqrt{3}} [(3m_\pi^2 - 4m_K^2) \cos \theta_\eta - 2\sqrt{2}m_K^2 \sin \theta_\eta] \quad (C.38)$$

$$D_L^u(\eta) = \frac{1}{4\sqrt{3}} m_\pi^2 (\cos \theta_\eta - \sqrt{2} \sin \theta_\eta), \quad (C.39)$$

and for $P = \eta'$

$$C(\eta') = \frac{1}{\sqrt{6}} (\sqrt{2} \sin \theta_\eta - \cos \theta_\eta) \quad (C.40)$$

$$D_L^d(\eta') = \frac{1}{4\sqrt{3}} [(3m_\pi^2 - 4m_K^2) \sin \theta_\eta + 2\sqrt{2}m_K^2 \cos \theta_\eta] \quad (C.41)$$

$$D_L^u(\eta') = \frac{1}{4\sqrt{3}} m_\pi^2 (\sin \theta_\eta + \sqrt{2} \cos \theta_\eta). \quad (C.42)$$

Here m_π and m_K are the masses of the neutral pion and Kaon, respectively, and θ_η is the $\eta - \eta'$ mixing angle.

In addition, $D_R^{d,u}(P) = -\left(D_L^{d,u}(P)\right)^*$.

Notice that the Wilson coefficients in Eq.(C.33) include all pseudoscalar and axial contributions to $\tau \rightarrow \ell P$. Therefore, this goes beyond some well-known results in the literature, see for example [94, 95], where box contributions were neglected.

Listing 24 TauLMeson.m

```

1 NameProcess = "TauLMeson";
2 NameObservables = {{BrTautoEPi, 2001, "BR(tau->e pi)"},
3   {BrTautoEEta, 2002, "BR(tau->e eta)"},
4   {BrTautoEEtap, 2003, "BR(tau->e eta')"},
5   {BrTautoMuPi, 2004, "BR(tau->mu pi)"},
6   {BrTautoMuEta, 2005, "BR(tau->mu eta)"},
7   {BrTautoMuEtap, 2006, "BR(tau->mu eta')"}}};
8

```

```

9 NeededOperators = {OllddSLL, OllddSRR, OllddSRL, OllddSLR,
10   OllddVRR, OllddVLL, OllddVRL, OllddVLR,
11   OlluuSLL, OlluuSRR, OlluuSRL, OlluuSLR,
12   OlluuVRR, OlluuVLL, OlluuVRL, OlluuVLR
13 };
14
15 Body = "TauLMeson.f90";

```

Listing 25 TauLMeson.f90

```

1 Real(dp) :: Fpi, thetaEta, mPi, mK, mEta, mEtap, meson_abs_T2, cont, &
2   & mP, CP, factor, BR
3 Complex(dp) :: BSLR, BSRL, BSRR, BVLL, BVLR, BVRL, BVR, &
4   & CSLL, CSLR, CSRL, CSRR, CVLL, CVLR, CVRL, CVRR, aP(2), bP(2), &
5   & DLdP, DRdP, DLuP, DRuP
6 Integer :: i1, i2, out, k1, k2
7
8 ! -----
9 ! tau -> l meson
10 ! Observable implemented by W. Porod, F. Staub and A. Vicente
11 ! Generalizes the analytical expressions in
12 ! E. Arganda et al, JHEP 0806 (2008) 079 [arXiv:0803.2039]
13 ! -----
14
15 Fpi=0.0924_dp! Pion decay constant in GeV
16 thetaEta=Pi/10._dp! eta-eta' mixing angle
17 mPi=0.13497_dp! Pion mass in GeV
18 mK=0.49761_dp! Kaon mass in GeV
19 mEta=0.548_dp! Eta mass in GeV
20 mEtap=0.958_dp! Eta' mass in GeV
21
22 !Mesons:
23 !1:Pi0
24 !2:Eta
25 !3:Eta'
26 Do i1=1,3
27   If (i1.eq.1) Then !1:Pi0
28     mP = mPi
29     CP = 1._dp
30     DLdP = - mPi**2/4._dp
31     DRdP = - Conjg(DLdP)
32     DLuP = mPi**2/4._dp
33     DRuP = - Conjg(DLuP)
34   Else If (i1.eq.2) Then !2:Eta
35     mP = mEta
36     CP = (Sin(thetaEta)+Sqrt(2._dp)*Cos(thetaEta))/Sqrt(6._dp)
37     DLdP = 1._dp/(4._dp*Sqrt(3._dp))*((3._dp*mPi**2-4._dp*mK**2) &
38       & *Cos(thetaEta)-2._dp*Sqrt(2._dp)*mK**2*Sin(thetaEta))
39     DRdP = - Conjg(DLdP)
40     DLuP = 1._dp/(4._dp*Sqrt(3._dp))*mPi**2*(Cos(thetaEta) &
41       & -Sqrt(2._dp)*Sin(thetaEta))
42     DRuP = - Conjg(DLuP)
43   Else If (i1.eq.3) Then !3:Eta'
44     mP = mEtap
45     CP = (Sqrt(2._dp)*Sin(thetaEta)-Cos(thetaEta))/Sqrt(6._dp)
46     DLdP = 1._dp/(4._dp*Sqrt(3._dp))*((3._dp*mPi**2-4._dp*mK**2) &
47       & *Sin(thetaEta)+2._dp*Sqrt(2._dp)*mK**2*Cos(thetaEta))
48     DRdP = - Conjg(DLdP)
49     DLuP = 1._dp/(4._dp*Sqrt(3._dp))*mPi**2*(Sin(thetaEta)+ &
50       & Sqrt(2._dp)*Cos(thetaEta))
51     DRuP = - Conjg(DLuP)
52   End If
53
54 !Leptons:
55 !1:e
56 !2:mu
57 Do i2=1,2
58   If (i2.eq.1) Then ! tau -> e P

```



```

59 out = 1
60 Elseif (i2.eq.2) Then      ! tau -> mu P
61 out = 2
62 End if
63
64 ! d-quark coefficients
65
66 BSLL = OllddSLL(3,out,1,1)
67 BSLR = OllddSLR(3,out,1,1)
68 BSRL = OllddSRL(3,out,1,1)
69 BSRR = OllddSRR(3,out,1,1)
70 BVLL = OllddVLL(3,out,1,1)
71 BVLR = OllddVLR(3,out,1,1)
72 BVRL = OllddVRL(3,out,1,1)
73 BVRR = OllddVRR(3,out,1,1)
74
75 ! u-quark coefficients
76
77 CSLL = OlluuSLL(3,out,1,1)
78 CSLR = OlluuSLR(3,out,1,1)
79 CSRL = OlluuSRL(3,out,1,1)
80 CSRR = OlluuSRR(3,out,1,1)
81 CVLL = OlluuVLL(3,out,1,1)
82 CVLR = OlluuVLR(3,out,1,1)
83 CVRL = OlluuVRL(3,out,1,1)
84 CVRR = OlluuVRR(3,out,1,1)
85
86 ! aP, bP scalar
87 aP(1) = Fpi/2._dp*(DLdP/mf_d(1)*(BSLL+BSRL) + DRdP/mf_d(1)*(BSLR+BSRR) &
88 & + DLuP/mf_u(1)*(CSLL+CSRL) + DRuP/mf_u(1)*(CSLR+CSRR))
89 bP(1) = Fpi/2._dp*(DLdP/mf_d(1)*(BSRL-BSLL) + DRdP/mf_d(1)*(BSRR-BSLR) &
90 & + DLuP/mf_u(1)*(CSRL-CSLL) + DRuP/mf_u(1)*(CSRR-CSLR))
91
92 ! aP, bP vector
93 aP(2) = Fpi/4._dp*CP*(mf_l(3)-mf_l(out))*(-BVLL+BVLR-BVRL+BVRR+ &
94 & CVLL-CVLR+CVRL+CVRR)
95 bP(2) = Fpi/4._dp*CP*(mf_l(3)+mf_l(out))*(-BVLL+BVLR+BVRL+BVRR+ &
96 & CVLL-CVLR-CVRL+CVRR)
97
98 ! averaged squared amplitude
99 meson_abs_T2=0._dp
100 Do k1=1,2
101 Do k2=1,2
102 cont=2._dp*mf_l(out)*mf_l(3)*(aP(k1)*conjg(aP(k2)) &
103 & -bP(k1)*conjg(bP(k2)))+ &
104 & (mf_l(3)**2+mf_l(out)**2-mP**2)*(aP(k1)*conjg(aP(k2))+ &
105 & bP(k1)*conjg(bP(k2)))
106 meson_abs_T2=meson_abs_T2+cont
107 End Do
108 End Do
109 meson_abs_T2=meson_abs_T2/(2._dp*mf_l(3))
110
111 ! branching ratio
112 factor=oo4pi*Sqrt(lamb(mf_l(3)**2,mf_l(out)**2,mP**2)) &
113 & /(mf_l(3)**2*GammaTau)*0.5_dp
114 BR=factor*meson_abs_T2
115 If (i1.eq.1) Then !pi
116 If (i2.eq.1) Then
117 BrTautoEPi = BR
118 Else
119 BrTautoMuPi = BR
120 End If
121 Elseif (i1.eq.2) Then !eta
122 If (i2.eq.1) Then
123 BrTautoEEta = BR
124 Else
125 BrTautoMuEta = BR
126 End If
127 Else !eta'

```

```

128   If (i2.eq.1) Then
129     BrTautoEEtap = BR
130   Else
131     BrTautoMuEtap = BR
132   End If
133 End if
134
135 End Do
136 End Do
137
138 Contains
139
140 Real(dp) Function lamb(x,y,z)
141 Real(dp), Intent(in) :: x,y,z
142   lamb=(x+y-z)**2-4._dp*x*y
143 End Function lamb

```

C.1.5: $h \rightarrow \ell_\alpha \ell_\beta$

The decay width is given by [96]

$$\begin{aligned}
 \Gamma(h \rightarrow \ell_\alpha \ell_\beta) &\equiv \Gamma(h \rightarrow \ell_\alpha \bar{\ell}_\beta) + \Gamma(h \rightarrow \bar{\ell}_\alpha \ell_\beta) = \\
 &\frac{1}{16\pi m_h} \left[\left(1 - \left(\frac{m_{\ell_\alpha} + m_{\ell_\beta}}{m_h} \right)^2 \right) \left(1 - \left(\frac{m_{\ell_\alpha} - m_{\ell_\beta}}{m_h} \right)^2 \right) \right]^{1/2} \\
 &\times \left[(m_h^2 - m_{\ell_\alpha}^2 - m_{\ell_\beta}^2) (|S_L|^2 + |S_R|^2)_{\alpha\beta} - 4m_{\ell_\alpha} m_{\ell_\beta} \text{Re}(S_L S_R^*)_{\alpha\beta} \right] \\
 &+ (\alpha \leftrightarrow \beta)
 \end{aligned} \tag{C.43}$$

Listing 26 hLLp.m

```

1 NameProcess = "hLLp";
2 NameObservables = {{BrhtoMuE, 1101, "BR(h->e mu)"},
3                    {BrhtoTauE, 1102, "BR(h->e tau)"},
4                    {BrhtoTauMu, 1103, "BR(h->mu tau)"};
5
6 NeededOperators = {OH2lSL, OH2lSR};
7
8 Body = "hLLp.f90";

```

Listing 27 hLLp.f90

```

1 Real(dp) :: width1, width2, width, mh, gamh, kinfactor
2 Complex(dp) :: SL1, SR1, SL2, SR2
3 Integer :: i1, gt1, gt2, hLoc
4
5 ! _____
6 ! h -> l l'
7 ! Observable implemented by W. Porod, F. Staub and A. Vicente
8 ! Based on E. Arganda et al, PRD 71 (2005) 035011 [hep-ph/0407302]
9 ! _____
10
11 !! NEXT LINE HAVE TO BE PARSED BY SARAH
12 ! Checking if there are several generations of Scalars and what is the SM-like doublet
13 @ If [getGen[HiggsBoson]>1, "hLoc = ↵
14     ↵ MaxLoc(Abs("<>ToString[HiggsMixingMatrix]<>"(2,:)),1)", "hLoc = 1"]
15 @ "mh = "<>ToString[SPhenoMass[HiggsBoson]]<> If [getGen[HiggsBoson]>1, "(hLoc)", ""]
16
17 @ "gamh = "<>ToString[SPhenoWidth[HiggsBoson]]<> If [getGen[HiggsBoson]>1, "(hLoc)", ""]
18
19 If (.not.L_BR) gamh = 4.5E-3_dp ! Decays not calculated; using SM value

```

```

20
21 Do i1=1,3
22
23 If (i1.eq.1) Then          ! h -> e mu
24   gt1 = 1
25   gt2 = 2
26 Elseif (i1.eq.2) Then     ! h -> e tau
27   gt1 = 1
28   gt2 = 3
29 Else                       ! h -> mu tau
30   gt1 = 2
31   gt2 = 3
32 End if
33
34 ! width = Gamma(h -> \bar{l1} l2) + Gamma(h -> l1 \bar{l2})
35
36 SL1 = OH2lSL(gt1,gt2,hLoc)
37 SR1 = OH2lSR(gt1,gt2,hLoc)
38 SL2 = OH2lSL(gt2,gt1,hLoc)
39 SR2 = OH2lSR(gt2,gt1,hLoc)
40
41 kinfactor = (1-(mf_l(gt1)+mf_l(gt2)/mh)**2)*&
42             & (1-(mf_l(gt1)-mf_l(gt2)/mh)**2)
43
44 width1 = (mh**2-mf_l(gt1)**2-mf_l(gt2)**2)*(Abs(SL1)**2+Abs(SR1)**2) &
45           & - 4._dp*mf_l(gt1)*mf_l(gt2)*Real(SL1*Conjg(SR1),dp)
46 width2 = (mh**2-mf_l(gt1)**2-mf_l(gt2)**2)*(Abs(SL2)**2+Abs(SR2)**2) &
47           & - 4._dp*mf_l(gt1)*mf_l(gt2)*Real(SL2*Conjg(SR2),dp)
48
49 ! decay width
50 width = ool6pi/mh * sqrt(kinfactor) * (width1+width2)
51
52 If (i1.eq.1) Then
53 BrhtoMuE = width/(width+gamh)
54 Elseif (i1.eq.2) Then
55 BrhtoTauE = width/(width+gamh)
56 Else
57 BrhtoTauMu = width/(width+gamh)
58 End if
59
60 End do

```

C.1.6: $Z \rightarrow \ell \alpha \ell \beta$

The decay width is given by [97]

$$\begin{aligned}
 \Gamma(Z \rightarrow \ell \alpha \ell \beta) &\equiv \Gamma(Z \rightarrow \ell \alpha \bar{\ell} \beta) + \Gamma(Z \rightarrow \bar{\ell} \alpha \ell \beta) = \\
 &\frac{m_Z}{48\pi} \left[2 \left(|R_1^L|^2 + |R_1^R|^2 \right) + \frac{m_Z^2}{4} \left(|R_2^L|^2 + |R_2^R|^2 \right) \right],
 \end{aligned} \tag{C.44}$$

where the charged lepton masses have been neglected.

Listing 28 ZLLp.m

```

1 NameProcess = "ZLLp";
2 NameObservables = {{BrZtoMuE, 1001, "BR(Z->e mu)"},
3                   {BrZtoTauE, 1002, "BR(Z->e tau)"},
4                   {BrZtoTauMu, 1003, "BR(Z->mu tau)"};
5
6 NeededOperators = {OZ2lSL, OZ2lSR, OZ2lVL, OZ2lVR};
7
8 Body = "ZLLp.f90";

```

Listing 29 ZLLp.f90

```

1 Real(dp) :: width
2 Integer :: i1, gt1, gt2
3
4 ! -----
5 ! Z -> l l'
6 ! Observable implemented by W. Porod, F. Staub and A. Vicente
7 ! Based on X. -J. Bi et al, PRD 63 (2001) 096008 [hep-ph/0010270]
8 ! -----
9
10 Do i1=1,3
11
12   If (i1.eq.1) Then           ! Z -> e mu
13     gt1 = 1
14     gt2 = 2
15   Elseif (i1.eq.2) Then      ! Z -> e tau
16     gt1 = 1
17     gt2 = 3
18   Else                       ! Z -> mu tau
19     gt1 = 2
20     gt2 = 3
21   End if
22
23   ! decay width
24   width = oo48pi*(2*(Abs(OZ2lVL(gt1,gt2))**2 +           &
25     & Abs(OZ2lVR(gt1,gt2))**2)*mZ                        &
26     & + (Abs(OZ2lSL(gt1,gt2))**2+Abs(OZ2lSR(gt1,gt2))**2) &
27     & * mZ * mZ2 * 0.25_dp)
28
29   If (i1.eq.1) Then
30     BrZtoMuE = width/(width+gamZ)
31   Elseif (i1.eq.2) Then
32     BrZtoTauE = width/(width+gamZ)
33   Else
34     BrZtoTauMu = width/(width+gamZ)
35   End if
36
37 End do

```

C.2: Quark flavor observables

QFV has been observed and its description in the SM due to the CKM matrix is well established. However, the large majority of BSM models causes additional contributions which have to be studied carefully, see for instance Refs. [98–122].

We give also here a description of the implementation of the different observables using the operators present in the **SPheno** output of **SARAH**.

C.3: $B_{s,d}^0 \rightarrow \ell^+ \ell^-$

Our analytical results for $B_{s,d}^0 \rightarrow \ell^+ \ell^-$ follow [103]. The $B^0 \equiv B_{s,d}^0$ decay width to a pair of charged leptons can be written as

$$\Gamma(B^0 \rightarrow \ell_\alpha^+ \ell_\beta^-) = \frac{|\mathcal{M}_{B\ell\ell}|^2}{16\pi M_B} \left[\left(1 - \left(\frac{m_{\ell_\alpha} + m_{\ell_\beta}}{m_B} \right)^2 \right) \left(1 - \left(\frac{m_{\ell_\alpha} - m_{\ell_\beta}}{m_B} \right)^2 \right) \right]^{1/2}. \quad (\text{C.45})$$

Here

$$\begin{aligned}
|\mathcal{M}_{B\ell\ell}|^2 = & 2|F_S|^2 \left[m_B^2 - (m_{\ell_\alpha} + m_{\ell_\beta})^2 \right] + 2|F_P|^2 \left[m_B^2 - (m_{\ell_\alpha} - m_{\ell_\beta})^2 \right] \\
& + 2|F_V|^2 \left[m_B^2 (m_{\ell_\alpha} - m_{\ell_\beta})^2 - (m_{\ell_\alpha}^2 - m_{\ell_\beta}^2)^2 \right] \\
& + 2|F_A|^2 \left[m_B^2 (m_{\ell_\alpha} + m_{\ell_\beta})^2 - (m_{\ell_\alpha}^2 - m_{\ell_\beta}^2)^2 \right] \\
& + 4\text{Re}(F_S F_V^*) (m_{\ell_\alpha} - m_{\ell_\beta}) \left[m_B^2 + (m_{\ell_\alpha} + m_{\ell_\beta})^2 \right] \\
& + 4\text{Re}(F_P F_A^*) (m_{\ell_\alpha} + m_{\ell_\beta}) \left[m_B^2 - (m_{\ell_\alpha} - m_{\ell_\beta})^2 \right], \tag{C.46}
\end{aligned}$$

and the F_X coefficients are defined in terms of our Wilson coefficients as¹²

$$F_S = \frac{i}{4} \frac{m_B^2 f_B}{m_d + m_{d'}} \left(E_{LL}^S + E_{LR}^S - E_{RR}^S - E_{RL}^S \right) \tag{C.47}$$

$$F_P = \frac{i}{4} \frac{m_B^2 f_B}{m_d + m_{d'}} \left(-E_{LL}^S + E_{LR}^S - E_{RR}^S + E_{RL}^S \right) \tag{C.48}$$

$$F_V = -\frac{i}{4} f_B \left(E_{LL}^V + E_{LR}^V - E_{RR}^V - E_{RL}^V \right) \tag{C.49}$$

$$F_A = -\frac{i}{4} f_B \left(-E_{LL}^V + E_{LR}^V - E_{RR}^V + E_{RL}^V \right), \tag{C.50}$$

where $f_B \equiv f_{B_{d,s}^0}$ is the $B_{d,s}^0$ decay constant and $m_{d,d'}$ are the masses of the quarks contained in the B meson, $B_d^0 \equiv \bar{b}d$ and $B_s^0 \equiv \bar{b}s$. In the lepton flavor conserving case, $\alpha = \beta$, the F_V contribution vanishes. In this case, the results in [103] are in agreement with previous computations [123, 124].

Listing 30 B0ll.m

```

1 NameProcess = "B0toLL ";
2 NameObservables = { {BrB0dEE, 4000, "BR(B^0_d->e e)"},
3                     {ratioB0dEE, 4001, "BR(B^0_d->e e)/BR(B^0_d->e e)_SM"},
4                     {BrB0sEE, 4002, "BR(B^0_s->e e)"},
5                     {ratioB0sEE, 4003, "BR(B^0_s->e e)/BR(B^0_s->e e)_SM"},
6                     {BrB0dMuMu, 4004, "BR(B^0_d->mu mu)"},
7                     {ratioB0dMuMu, 4005, "BR(B^0_d->mu mu)/BR(B^0_d->mu mu)_SM"},
8                     {BrB0sMuMu, 4006, "BR(B^0_s->mu mu)"},
9                     {ratioB0sMuMu, 4007, "BR(B^0_s->mu mu)/BR(B^0_s->mu mu)_SM"},
10                    {BrB0dTauTau, 4008, "BR(B^0_d->tau tau)"},
11                    {ratioB0dTauTau, 4009, "BR(B^0_d->tau tau)/BR(B^0_d->tau tau)_SM"},
12                    {BrB0sTauTau, 4010, "BR(B^0_s->tau tau)"},
13                    {ratioB0sTauTau, 4011, "BR(B^0_s->tau tau)/BR(B^0_s->tau tau)
14                      <-> tau)_SM"} };
15
16 NeededOperators = {OddllSLL, OddllSRR, OddllSRL, OddllSLR,
17                   OddllVRR, OddllVLL, OddllVRL, OddllVLR,
18                   OddllSLLSM, OddllSRRSM, OddllSRLSM, OddllSLRSM,
19                   OddllVRRSM, OddllVLLSM, OddllVRLSM, OddllVLRSM};
20
21 Body = "B0ll.f90";

```

Listing 31 B0ll.f90

```

1 Real(dp) :: AmpSquared, AmpSquared2, AmpSquared_SM, AmpSquared2_SM, &
2           & width_SM, width
3 Real(dp) :: MassB0s, MassB0d, fBs, fBd, TauB0s, TauB0d
4 Real(dp) :: hbar=6.58211899E-25_dp
5 Real(dp) :: MassB0, MassB02, fB0, GammaB0

```

¹²Notice that our effective Lagrangian differs from the one in [103] by a $1/(4\pi)^2$ factor. This relative factor has been absorbed in the expression for $\mathcal{M}_{B\ell\ell}$, see Eq.(C.46).

```

6 Complex(dp) :: CS(4), CV(4), CT(4)
7 Complex(dp) :: FS=0._dp, FP=0._dp, FV=0._dp, FA=0._dp
8 Integer :: i1, gt1, gt2, gt3, gt4
9
10 ! -----
11 ! B0 -> l l
12 ! Observable implemented by W. Porod, F. Staub and A. Vicente
13 ! Based on A. Dedes et al, PRD 79 (2009) 055006 [arXiv:0812.4320]
14 ! -----
15
16 ! Using global hadronic data
17 fBd = f_B0d_CONST
18 fBs = f_B0s_CONST
19 TauB0d = tau_B0d
20 TauB0s = tau_B0s
21 MassB0d = mass_B0d
22 MassB0s = mass_B0s
23
24 Do i1=1,6
25   gt1 = 3
26   If (i1.eq.1) Then ! B0d -> e+ e-
27     MassB0 = MassB0d
28     MassB02 = MassB0d**2
29     fB0 = fBd
30     GammaB0 = (hbar)/(TauB0d)
31     gt2 = 1
32     gt3 = 1
33     gt4 = 1
34   Else if (i1.eq.2) Then ! B0s -> e+ e-
35     MassB0 = MassB0s
36     MassB02 = MassB0s**2
37     fB0 = fBs
38     GammaB0 = (hbar)/(TauB0s)
39     gt2 = 2
40     gt3 = 1
41     gt4 = 1
42   Else if (i1.eq.3) Then ! B0d -> mu+ mu-
43     MassB0 = MassB0d
44     MassB02 = MassB0d**2
45     fB0 = fBd
46     GammaB0 = (hbar)/(TauB0d)
47     gt2 = 1
48     gt3 = 2
49     gt4 = 2
50   Else if (i1.eq.4) Then ! B0s -> mu+ mu-
51     MassB0 = MassB0s
52     MassB02 = MassB0s**2
53     fB0 = fBs
54     GammaB0 = (hbar)/(TauB0s)
55     gt2 = 2
56     gt3 = 2
57     gt4 = 2
58   Else if (i1.eq.5) Then ! B0d -> tau+ tau-
59     MassB0 = MassB0d
60     MassB02 = MassB0d**2
61     fB0 = fBd
62     GammaB0 = (hbar)/(TauB0d)
63     gt2 = 1
64     gt3 = 3
65     gt4 = 3
66   Else if (i1.eq.6) Then ! B0s -> tau+ tau-
67     MassB0 = MassB0s
68     MassB02 = MassB0s**2
69     fB0 = fBs
70     GammaB0 = (hbar)/(TauB0s)
71     gt2 = 2
72     gt3 = 3
73     gt4 = 3
74   End if

```

```

75
76 ! BSM contributions
77
78 CS(1) = OddllSRR(gt1,gt2,gt3,gt4)
79 CS(2) = OddllSRL(gt1,gt2,gt3,gt4)
80 CS(3) = OddllSLL(gt1,gt2,gt3,gt4)
81 CS(4) = OddllSLR(gt1,gt2,gt3,gt4)
82
83 CV(1) = OddllVLL(gt1,gt2,gt3,gt4)
84 CV(2) = OddllVLR(gt1,gt2,gt3,gt4)
85 CV(3) = OddllVRR(gt1,gt2,gt3,gt4)
86 CV(4) = OddllVRL(gt1,gt2,gt3,gt4)
87
88 FS= 0.25_dp*MassB02*fB0/(MFd(gt1)+MFd(gt2))*( CS(1)+CS(2)-CS(3)-CS(4))
89 FP= 0.25_dp*MassB02*fB0/(MFd(gt1)+MFd(gt2))*(-CS(1)+CS(2)-CS(3)+CS(4))
90 FV= -0.25_dp*fB0*( CV(1)+CV(2)-CV(3)-CV(4))
91 FA= -0.25_dp*fB0*(-CV(1)+CV(2)-CV(3)+CV(4))
92
93 AmpSquared = 2 * abs(FS)**2 * (MassB02 - (mf_l(gt3)+mf_l(gt4))**2) &
94 & + 2 * abs(FP)**2 * (MassB02 - (mf_l(gt3)-mf_l(gt4))**2) &
95 & + 2 * abs(FV)**2 * (MassB02*(mf_l(gt4)-mf_l(gt3))**2 &
96 & - (mf_l2(gt4)-mf_l2(gt3))**2) &
97 & + 2 * abs(FA)**2 * (MassB02*(mf_l(gt4)+mf_l(gt3))**2 - &
98 & (mf_l2(gt4)-mf_l2(gt3))**2) &
99 & + 4 * REAL(FS*conjg(FV)) * (mf_l(gt3)-mf_l(gt4)) * (MassB02 &
100 & + (mf_l(gt3)+mf_l(gt4))**2) &
101 & + 4 * REAL(FP*conjg(FA)) * (mf_l(gt3)+mf_l(gt4)) * (MassB02 &
102 & - (mf_l(gt3)-mf_l(gt4))**2)
103
104 width = oo16pi * AmpSquared / MassB0 * &
105 & sqrt(1-((mf_l(gt4)+mf_l(gt3))/MassB0)**2) &
106 & * sqrt(1-((mf_l(gt4)-mf_l(gt3))/MassB0)**2)*(Alpha/Alpha_160)**4
107
108
109 ! SM contributions
110
111 CS(1) = OddllSRRSM(gt1,gt2,gt3,gt4)
112 CS(2) = OddllSRLSM(gt1,gt2,gt3,gt4)
113 CS(3) = OddllSLLSM(gt1,gt2,gt3,gt4)
114 CS(4) = OddllSLRSM(gt1,gt2,gt3,gt4)
115
116 CV(1) = OddllVLLSM(gt1,gt2,gt3,gt4)
117 CV(2) = OddllVLRSM(gt1,gt2,gt3,gt4)
118 CV(3) = OddllVRRSM(gt1,gt2,gt3,gt4)
119 CV(4) = OddllVRLSM(gt1,gt2,gt3,gt4)
120
121 FS= 0.25_dp*MassB02*fB0/(MFd(gt1)+MFd(gt2))*( CS(1)+CS(2)-CS(3)-CS(4))
122 FP= 0.25_dp*MassB02*fB0/(MFd(gt1)+MFd(gt2))*(-CS(1)+CS(2)-CS(3)+CS(4))
123 FV= -0.25_dp*fB0*( CV(1)+CV(2)-CV(3)-CV(4))
124 FA= -0.25_dp*fB0*(-CV(1)+CV(2)-CV(3)+CV(4))
125
126 AmpSquared = 2 * abs(FS)**2 * (MassB02 - (mf_l(gt3)+mf_l(gt4))**2) &
127 & + 2 * abs(FP)**2 * (MassB02 - (mf_l(gt3)-mf_l(gt4))**2) &
128 & + 2 * abs(FV)**2 * (MassB02*(mf_l(gt4)-mf_l(gt3))**2 - &
129 & (mf_l2(gt4)-mf_l2(gt3))**2) &
130 & + 2 * abs(FA)**2 * (MassB02*(mf_l(gt4)+mf_l(gt3))**2 - &
131 & (mf_l2(gt4)-mf_l2(gt3))**2) &
132 & + 4 * REAL(FS*conjg(FV)) * (mf_l(gt3)-mf_l(gt4)) * (MassB02 &
133 & + (mf_l(gt3)+mf_l(gt4))**2) &
134 & + 4 * REAL(FP*conjg(FA)) * (mf_l(gt3)+mf_l(gt4)) * (MassB02 &
135 & - (mf_l(gt3)-mf_l(gt4))**2)
136
137 width_SM = oo16pi * AmpSquared / MassB0 * sqrt(1-((mf_l(gt4)+ &
138 & mf_l(gt3))/MassB0)**2) &
139 & * sqrt(1-((mf_l(gt4)-mf_l(gt3))/MassB0)**2)*(Alpha/Alpha_160)**4
140
141
142 If (i1.Eq.1) Then
143 BrB0dEE= width / GammaB0

```

```

144 ratioB0dEE= width / width_SM
145 Else If (i1.Eq.2) Then
146   BrB0sEE= width / GammaB0
147   ratioB0sEE= width / width_SM
148 Else If (i1.Eq.3) Then
149   BrB0dMuMu= width / GammaB0
150   ratioB0dMuMu= width / width_SM
151 Else If (i1.Eq.4) Then
152   BrB0sMuMu= width / GammaB0
153   ratioB0sMuMu= width / width_SM
154 Else If (i1.Eq.5) Then
155   BrB0dTauTau= width / GammaB0
156   ratioB0dTauTau= width / width_SM
157 Else If (i1.Eq.6) Then
158   BrB0sTauTau= width / GammaB0
159   ratioB0sTauTau= width / width_SM
160 End If
161
162 End do

```

C.4: $\bar{B} \rightarrow X_s \gamma$

The branching ratio for $\bar{B} \rightarrow X_s \gamma$, with a cut $E_\gamma > 1.6$ GeV in the \bar{B} rest frame, can be obtained as [104, 125]

$$\text{BR}(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = 10^{-4} \left[a_{SM} + a_{77} \left(|\delta C_7^{(0)}|^2 + |\delta C_7'^{(0)}|^2 \right) + a_{88} \left(|\delta C_8^{(0)}|^2 + |\delta C_8'^{(0)}|^2 \right) + \text{Re} \left(a_7 \delta C_7^{(0)} + a_8 \delta C_8^{(0)} + a_{78} \left(\delta C_7^{(0)} \delta C_8^{(0)*} + \delta C_7'^{(0)} \delta C_8'^{(0)*} \right) \right) \right], \quad (\text{C.51})$$

where $a_{SM} = 3.15$ is the NNLO SM prediction [51, 126], the other a coefficients in Eq.(C.51) are found to be

$$\begin{aligned} a_{77} &= 4.743 \\ a_{88} &= 0.789 \\ a_7 &= -7.184 + 0.612 i \\ a_8 &= -2.225 - 0.557 i \\ a_{78} &= 2.454 - 0.884 i \end{aligned} \quad (\text{C.52})$$

and we have defined $\delta C_i^{(0)} = C_i^{(0)} - C_i^{(0) \text{ SM}}$. Finally, the $C_i^{(0)}$ coefficients can be written in terms of $Q_{1,2}^{L,R}$ in Eqs.(A.11) and (A.12) as

$$C_7^{(0)} = n_{CQ} Q_1^R \quad (\text{C.53})$$

$$C_7'^{(0)} = n_{CQ} Q_1^L \quad (\text{C.54})$$

$$C_8^{(0)} = n_{CQ} Q_2^R \quad (\text{C.55})$$

$$C_8'^{(0)} = n_{CQ} Q_2^L \quad (\text{C.56})$$

where $n_{CQ}^{-1} = -\frac{G_F}{4\sqrt{2}\pi^2} V_{tb} V_{ts}^*$ and V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

Listing 32 bsGamma.m

```

1 NameProcess = "bsGamma";
2 NameObservables = {{BrBsGamma, 200, "BR(B->X_s gamma)"},
3   {ratioBsGamma, 201, "BR(B->X_s gamma)/BR(B->X_s gamma)_SM"}};
4
5 NeededOperators = {CC7, CC7p, CC8, CC8p,
6   CC7SM, CC7pSM, CC8SM, CC8pSM};
7
8 Body = "bsGamma.f90 ";

```


Listing 33 bsGamma.f90

```

1 Integer :: gt1, gt2
2 Complex(dp) :: norm, delta_C7_0, delta_C7p_0, delta_C8_0, delta_C8p_0
3 Real(dp) :: NNLO_SM
4
5 ! -----
6 ! \bar{B} -> X_s gamma (Egamma > 1.6 GeV)
7 ! Observable implemented by W. Porod, F. Staub and A. Vicente
8 ! Based on E. Lunghi, J. Matias, JHEP 0704 (2007) 058 [hep-ph/0612166]
9 ! -----
10
11 gt1=3 !b
12 gt2=2 !s
13
14 ! normalization of our Wilson coefficients
15 ! relative to the ones used in hep-ph/0612166
16 norm = -CKM_160(3,3)*Conjg(CKM_160(gt1,gt2))*Alpha_160/ &
17 & (8._dp*Pi*sinW2_160*mW2)
18
19 ! Wilson coefficients
20 delta_C7_0 =(CC7(gt1,gt2)-CC7SM(gt1,gt2))/norm
21 delta_C7p_0=(CC7p(gt1,gt2)-CC7pSM(gt1,gt2))/norm
22 delta_C8_0 =(CC8(gt1,gt2)-CC8SM(gt1,gt2))/norm
23 delta_C8p_0=(CC8p(gt1,gt2)-CC8pSM(gt1,gt2))/norm
24
25 ! NNLO SM prediction
26 ! as obtained in M. Misiak et al, PRL 98 (2007) 022002
27 ! and M. Misiak and M. Steinhauser, NPB 764 (2007) 62
28 NNLO_SM=3.15_dp
29
30 BrBsGamma=NNLO_SM+4.743_dp*(Abs(delta_C7_0)**2+Abs(delta_C7p_0)**2)&
31 &+0.789_dp*(Abs(delta_C8_0)**2+Abs(delta_C8p_0)**2)&
32 &+Real((-7.184_dp,0.612_dp)*delta_C7_0&
33 &+(-2.225_dp,-0.557_dp)*delta_C8_0+(2.454_dp,-0.884_dp)*&
34 &(delta_C7_0*conjg(delta_C8_0)+delta_C7p_0*conjg(delta_C8p_0)),dp)
35
36 ! ratio BSM/SM
37 ratioBsGamma = BrBsGamma/NNLO_SM
38
39 ! branching ratio
40 BrBsGamma=1E-4_dp*BrBsGamma

```

C.5: $\bar{B} \rightarrow X_s \ell^+ \ell^-$

Our results for $\bar{B} \rightarrow X_s \ell^+ \ell^-$ are based on [106], expanded with the addition of prime operators contributions [127]. The branching ratios for the $\ell = e$ case can be written as

$$\begin{aligned}
10^7 \text{BR}(\bar{B} \rightarrow X_s e^+ e^-) = & 2.3148 - 0.001658 \text{Im}(R_{10}) + 0.0005 \text{Im}(R_{10} R_8^* + R_{10}' R_8'^*) \\
& + 0.0523 \text{Im}(R_7) + 0.02266 \text{Im}(R_7 R_8^* + R_7' R_8'^*) + 0.00496 \text{Im}(R_7 R_9^* + R_7' R_9'^*) \\
& + 0.00518 \text{Im}(R_8) + 0.0261 \text{Im}(R_8 R_9^* + R_8' R_9'^*) - 0.00621 \text{Im}(R_9) - 0.5420 \text{Re}(R_{10}) \\
& - 0.03340 \text{Re}(R_7) + 0.0153 \text{Re}(R_7 R_{10}^* + R_7' R_{10}'^*) + 0.0673 \text{Re}(R_7 R_8^* + R_7' R_8'^*) \\
& - 0.86916 \text{Re}(R_7 R_9^* + R_7' R_9'^*) - 0.0135 \text{Re}(R_8) + 0.00185 \text{Re}(R_8 R_{10} + R_8' R_{10}'^*) \\
& - 0.09921 \text{Re}(R_8 R_9^* + R_8' R_9'^*) + 2.833 \text{Re}(R_9) - 0.10698 \text{Re}(R_9 R_{10}^* + R_9' R_{10}'^*) \\
& + 11.0348 (|R_{10}|^2 + |R_{10}'|^2) + 0.2804 (|R_7|^2 + |R_7'|^2) \\
& + 0.003763 (|R_8|^2 + |R_8'|^2) + 1.527 (|R_9|^2 + |R_9'|^2), \tag{C.57}
\end{aligned}$$

whereas for the $\ell = \mu$ case one gets

$$\begin{aligned}
10^7 \text{BR}(\bar{B} \rightarrow X_s \mu^+ \mu^-) = & 2.1774 - 0.001658 \text{Im}(R_{10}) + 0.0005 \text{Im}(R_{10} R_8^* + R_{10}' R_8'^*) \\
& + 0.0534 \text{Im}(R_7) + 0.02266 \text{Im}(R_7 R_8^* + R_7' R_8'^*) + 0.00496 \text{Im}(R_7 R_9^* + R_7' R_9'^*) \\
& + 0.00527 \text{Im}(R_8) + 0.0261 \text{Im}(R_8 R_9^* + R_8' R_9'^*) - 0.0115 \text{Im}(R_9) - 0.5420 \text{Re}(R_{10}) \\
& + 0.0208 \text{Re}(R_7) + 0.0153 \text{Re}(R_7 R_{10}^* + R_7' R_{10}'^*) + 0.0648 \text{Re}(R_7 R_8^* + R_7' R_8'^*) \\
& - 0.8545 \text{Re}(R_7 R_9^* + R_7' R_9'^*) - 0.00938 \text{Re}(R_8) + 0.00185 \text{Re}(R_8 R_{10} + R_8' R_{10}') \\
& - 0.0981 \text{Re}(R_8 R_9^* + R_8' R_9'^*) + 2.6917 \text{Re}(R_9) - 0.10698 \text{Re}(R_9 R_{10}^* + R_9' R_{10}'^*) \\
& + 10.7652 \left(|R_{10}|^2 + |R_{10}'|^2 \right) + 0.2880 \left(|R_7|^2 + |R_7'|^2 \right) \\
& + 0.003763 \left(|R_8|^2 + |R_8'|^2 \right) + 1.527 \left(|R_9|^2 + |R_9'|^2 \right). \tag{C.58}
\end{aligned}$$

Here we have defined the ratios of Wilson coefficients

$$R_{7,8} = \frac{Q_{1,2}^R}{Q_{1,2}^{R,\text{SM}}} \quad , \quad R_{7,8}' = \frac{Q_{1,2}^L}{Q_{1,2}^{L,\text{SM}}} \tag{C.59}$$

as well as

$$R_{9,10} = \frac{E_{LL}^V \pm E_{LR}^V}{E_{LL}^{V,\text{SM}} \pm E_{LR}^{V,\text{SM}}} \quad , \quad R_{9,10}' = \frac{E_{RR}^V \pm E_{RL}^V}{E_{RR}^{V,\text{SM}} \pm E_{RL}^{V,\text{SM}}} . \tag{C.60}$$

Listing 34 BtoSLL.m

```

1 NameProcess = "BtoSLL ";
2 NameObservables = {{BrBtoSEE, 5000, "BR(B-> s e e)"},
3   {ratioBtoSEE, 5001, "BR(B-> s e e)/BR(B-> s e e)_SM"},
4   {BrBtoSMuMu, 5002, "BR(B-> s mu mu)"},
5   {ratioBtoSMuMu, 5003, "BR(B-> s mu mu)/BR(B-> s mu mu)_SM"}};
6
7 NeededOperators = {OddllVRR, OddllVLL, OddllVRL, OddllVLR,
8   CC7, CC7p, CC8, CC8p,
9   OddllVRRSM, OddllVLLSM, OddllVRLSM, OddllVLRSM,
10  CC7SM, CC7pSM, CC8SM, CC8pSM
11 };
12
13 Body = "BtoSLL.f90";

```

Listing 35 BtoSLL.f90

```

1 Complex(dp) :: c7(2), c7p(2), c8(2), c8p(2), r7, r7p, r8, r8p, norm, &
2   & r9(2), r9p(2), r10(2), r10p(2), &
3   & c9ee(2), c9pee(2), c10ee(2), c10pee(2), &
4   & c9_ee(2), c9p_ee(2), c10_ee(2), c10p_ee(2), &
5   & c9mm(2), c9pmm(2), c10mm(2), c10pmm(2), c9_cmm(2), &
6   & c9p_cmm(2), c10_cmm(2), c10p_cmm(2)
7
8 ! -----
9 ! \bar{B} -> X_s l+ l-
10 ! Observable implemented by W. Porod, F. Staub and A. Vicente
11 ! Based on T. Huber et al, NPB 740 (2006) 105, [hep-ph/0512066]
12 ! Prime operators added after private communication with E. Lunghi
13 ! -----
14
15 ! Wilson coefficients
16
17 c7(1) = CC7(3,2)
18 c7(2) = CC7SM(3,2)
19 c7p(1) = CC7p(3,2)
20 c7p(2) = CC7pSM(3,2)
21
22 c8(1) = CC8(3,2)

```

```

23 c8(2) = CC8SM(3,2)
24 c8p(1) = CC8p(3,2)
25 c8p(2) = CC8pSM(3,2)
26
27 c9ee(1) = OddllVLL(3,2,1,1)+OddllVLR(3,2,1,1)
28 c9ee(2) = (OddllVLLSM(3,2,1,1)+OddllVLRSM(3,2,1,1))
29 c9mm(1) = OddllVLL(3,2,2,2)+OddllVLR(3,2,2,2)
30 c9mm(2) = (OddllVLLSM(3,2,2,2)+OddllVLRSM(3,2,2,2))
31 c9pee(1) = OddllVRR(3,2,1,1)+OddllVRL(3,2,1,1)
32 c9pee(2) = (OddllVRRSM(3,2,1,1)+OddllVRLSM(3,2,1,1))
33 c9pmm(1) = OddllVRR(3,2,2,2)+OddllVRL(3,2,2,2)
34 c9pmm(2) = (OddllVRRSM(3,2,2,2)+OddllVRLSM(3,2,2,2))
35
36 c10ee(1) = OddllVLL(3,2,1,1)-OddllVLR(3,2,1,1)
37 c10ee(2) = (OddllVLLSM(3,2,1,1)-OddllVLRSM(3,2,1,1))
38 c10mm(1) = OddllVLL(3,2,2,2)-OddllVLR(3,2,2,2)
39 c10mm(2) = (OddllVLLSM(3,2,2,2)-OddllVLRSM(3,2,2,2))
40 c10pee(1) = OddllVRR(3,2,1,1)-OddllVRL(3,2,1,1)
41 c10pee(2) = (OddllVRRSM(3,2,1,1)-OddllVRLSM(3,2,1,1))
42 c10pmm(1) = OddllVRR(3,2,2,2)-OddllVRL(3,2,2,2)
43 c10pmm(2) = (OddllVRRSM(3,2,2,2)-OddllVRLSM(3,2,2,2))
44
45 ! ratios
46
47 r7 = c7(1) / c7(2)
48 r7p = c7p(1) / c7(2)
49 r8 = c8(1) / c8(2)
50 r8p = c8p(1) / c8(2)
51
52 r9(1) = c9ee(1)/c9ee(2)
53 r9(2) = c9mm(1)/c9mm(2)
54 r9p(1) = c9pee(1)/c9ee(2)
55 r9p(2) = c9pmm(1)/c9mm(2)
56
57 r10(1) = c10ee(1)/c10ee(2)
58 r10(2) = c10mm(1)/c10mm(2)
59 r10p(1) = c10pee(1)/c10ee(2)
60 r10p(2) = c10pmm(1)/c10mm(2)
61
62 BrBtoSEE = (2.3148_dp - 1.658e-3_dp * Aimag(R10(1)) &
63 & + 5.e-4_dp * Aimag(r10(1)*Conjg(r8) + r10p(1)*Conjg(r8p) ) &
64 & + 5.23e-2_dp * Aimag(r7) + 5.18e-3_dp * Aimag(r8) &
65 & + 2.266e-2_dp * Aimag(r7 * Conjg(r8) + r7p * Conjg(r8p) ) &
66 & + 4.96e-3_dp * Aimag(r7 * Conjg(r9(1)) + r7p * Conjg(r9p(1)) ) &
67 & + 2.61e-2_dp * Aimag(r8 * Conjg(r9(1)) + r8p * Conjg(r9p(1)) ) &
68 & - 6.21e-3_dp * Aimag(r9(1)) - 0.5420_dp * Real( r10(1), dp) &
69 & - 3.340e-2_dp * Real(r7, dp) - 1.35e-2_dp * Real(r8, dp) &
70 & + 1.53e-2_dp * Real(r7*Conjg(r10(1)) + r7p*Conjg(r10p(1)), dp ) &
71 & + 6.73e-2_dp * Real(r7 * Conjg(r8) + r7p * Conjg(r8p), dp ) &
72 & - 0.86916_dp * Real(r7*Conjg(r9(1)) + r7p*Conjg(r9p(1)), dp ) &
73 & + 1.85e-3_dp * Real(r8*Conjg(r10(1)) + r8p*Conjg(r10p(1)), dp ) &
74 & - 9.921e-2_dp * Real(r8*Conjg(r9(1)) + r8p*Conjg(r9p(1)), dp ) &
75 & + 2.833_dp * Real(r9(1), dp) + 0.2804_dp * (Abs(r7)**2 + Abs(r7p)**2)&
76 & - 0.10698_dp * Real( r9(1) * Conjg(r10(1)) &
77 & + r9p(1) * Conjg(r10p(1)), dp) &
78 & + 11.0348_dp * (Abs(r10(1))**2 + Abs(r10p(1))**2 ) &
79 & + 1.527_dp * (Abs(r9(1))**2 + Abs(r9p(1))**2 ) &
80 & + 3.763e-3_dp * (Abs(r8)**2 + Abs(r8p)**2 ) )
81
82 ! ratio BR(B -> Xs mu+ mu-)/BR(B -> Xs e+ e-)_SM
83 ratioBtoSee = BrBtoSEE/16.5529_dp
84
85 ! branching ratio B -> Xs e+ e-
86 BrBtoSEE = BrBtoSEE* 1.e-7_dp
87
88 BrBtoSMuMu = (2.1774_dp - 1.658e-3_dp * Aimag(R10(2)) &
89 & + 5.e-4_dp * Aimag(r10(2)*Conjg(r8) + r10p(2)*Conjg(r8p) ) &
90 & + 5.34e-2_dp * Aimag(r7) + 5.27e-3_dp * Aimag(r8) &
91 & + 2.266e-2_dp * Aimag(r7 * Conjg(r8) + r7p * Conjg(r8p) ) &

```

```

92 & + 4.96e-3_dp * Aimag(r7 * Conjg(r9(2)) + r7p * Conjg(r9p(2)) ) &
93 & + 2.61e-2_dp * Aimag(r8 * Conjg(r9(2)) + r8p * Conjg(r9p(2)) ) &
94 & - 1.15e-2_dp * Aimag(r9(2)) - 0.5420_dp * Real( r10(2), dp) &
95 & + 2.08e-2_dp * Real(r7,dp) - 9.38e-3_dp * Real(r8,dp) &
96 & + 1.53e-2_dp * Real(r7*Conjg(r10(2)) + r7p*Conjg(r10p(2)), dp ) &
97 & + 6.848e-2_dp * Real(r7 * Conjg(r8) + r7p * Conjg(r8p), dp ) &
98 & - 0.8545_dp * Real(r7*Conjg(r9(2)) + r7p*Conjg(r9p(2)), dp ) &
99 & + 1.85e-3_dp * Real(r8*Conjg(r10(2)) + r8p*Conjg(r10p(2)), dp ) &
100 & - 9.81e-2_dp * Real(r8*Conjg(r9(2)) + r8p*Conjg(r9p(2)), dp ) &
101 & + 2.6917_dp * Real(r9(2),dp) + 0.2880_dp*(Abs(r7)**2+Abs(r7p)**2) &
102 & - 0.10698_dp * Real( r9(2) * Conjg(r10(2)) &
103 & + r9p(2) * Conjg(r10p(2)), dp) &
104 & + 10.7652_dp * (Abs(r10(2))**2 + Abs(r10p(2))**2 ) &
105 & + 1.4884_dp * (Abs(r9(2))**2 + Abs(r9p(2))**2 ) &
106 & + 3.81e-3_dp * (Abs(r8)**2 + Abs(r8p)**2 ) &
107
108 ! ratio BR(B -> Xs mu+ mu-)/BR(B -> Xs mu+ mu-)_SM
109 ratioBtoSMuMu = BrBtoSMuMu/16.0479_dp
110
111 ! branching ratio B -> Xs mu+ mu-
112 BrBtoSMuMu = BrBtoSMuMu* 1.e-7_dp

```

C.6: $B^+ \rightarrow K^+ \ell^+ \ell^-$

Our results for $B^+ \rightarrow K^+ \ell^+ \ell^-$ are based on the expressions given in [102]. The branching ratio for $B^+ \rightarrow K^+ \mu^+ \mu^-$ in the high- q^2 region, q^2 being the dilepton invariant mass squared, can be written as

$$\text{BR} \left(B^+ \rightarrow K^+ \mu^+ \mu^- \right)_{q^2 \in [14.18, 22] \text{ GeV}^2} \simeq 1.11 + 0.22 \left(C_7^{\text{NP}} + C_7' \right) + 0.27 \left(C_9^{\text{NP}} + C_9' \right) - 0.27 \left(C_{10}^{\text{NP}} + C_{10}' \right). \quad (\text{C.61})$$

The coefficients in Eq. (C.61) can be related to the ones in our generic Lagrangian as

$$C_7^{\text{NP}} = n_{CQ} \left(Q_1^R - Q_1^{R,\text{SM}} \right) \quad (\text{C.62})$$

$$C_7' = n_{CQ} Q_1^L \quad (\text{C.63})$$

$$C_9^{\text{NP}} = n_{CQ} \left[\left(E_{LL}^V + E_{LR}^V \right) - \left(E_{LL}^{V,\text{SM}} + E_{LR}^{V,\text{SM}} \right) \right] \quad (\text{C.64})$$

$$C_9' = n_{CQ} \left(E_{RR}^V + E_{RL}^V \right) \quad (\text{C.65})$$

$$C_{10}^{\text{NP}} = n_{CQ} \left[\left(E_{LL}^V - E_{LR}^V \right) - \left(E_{LL}^{V,\text{SM}} - E_{LR}^{V,\text{SM}} \right) \right] \quad (\text{C.66})$$

$$C_{10}' = n_{CQ} \left(E_{RR}^V - E_{RL}^V \right) \quad (\text{C.67})$$

where the normalization factor n_{CQ} was already defined after Eq. (C.56).

Listing 36 BtoKLL.m

```

1 NameProcess = "BtoKLL";
2 NameObservables = {{BrBtoKmumu, 6000, "BR(B -> K mu mu)"},
3 {ratioBtoKmumu, 6001, "BR(B -> K mu mu)/BR(B -> K mu mu)_SM"}};
4
5 NeededOperators = {OddllVRR, OddllVLL, OddllVRL, OddllVLR, CC7, CC7p,
6 OddllVRRSM, OddllVLLSM, OddllVRLSM, OddllVLRSM, CC7SM, CC7pSM
7 };
8
9 Body = "BtoKLL.f90";

```

Listing 37 BtoKLL.f90

```

1 Complex(dp) :: c7NP, c7p, c9NP, c9p, c10NP, c10p, norm
2 Real(dp) :: GF

```

```

3 |
4 | ! -----
5 | ! B^+ -> K^+ l+ l- (14.18 GeV^2 < q^2 < 22 GeV^2)
6 | ! Observable implemented by W. Porod, F. Staub and A. Vicente
7 | ! Based on W. Altmannshofer, D. M. Straub, EPJ C 73 (2013) 2646
8 | ! [arXiv:1308.1501]
9 | ! -----
10 |
11 | c7NP = (CC7(3,2) - CC7SM(3,2))
12 | c7p = CC7p(3,2)
13 | c9NP = (OddllVLL(3,2,1,1)+OddllVLR(3,2,1,1) - &
14 |         & (OddllVLLSM(3,2,1,1)+OddllVLRSM(3,2,1,1)))
15 | c9p = (OddllVRR(3,2,1,1)+OddllVRL(3,2,1,1))
16 | c10NP = (OddllVLL(3,2,1,1)-OddllVLR(3,2,1,1) - &
17 |         & (OddllVLLSM(3,2,1,1)-OddllVLRSM(3,2,1,1)))
18 | c10p = (OddllVRR(3,2,1,1)-OddllVRL(3,2,1,1))
19 |
20 |
21 | ! running GF
22 | GF = (Alpha_160*4._dp*Pi/sinW2_160)/mw**2*sqrt2/8._dp
23 |
24 | ! normalization of our Wilson coefficients
25 | ! relative to the ones used in arXiv:1308.1501
26 | norm = - 0.016pi^2*4._dp*GF/sqrt2*CKM_160(3,3)*Conjg(CKM_160(3,2))
27 |
28 | ! Branching ratio in the high-q^2 region
29 | ! q^2 in [14.18,22] GeV^2
30 | BrBtoKmumu = (1.11_dp + 0.22_dp*(c7NP+c7p)/norm + &
31 |             & 0.27_dp*(c9NP+c9p)/norm - 0.27_dp*(c10NP+c10p)/norm)
32 |
33 | ! ratio relative to SM
34 | ratioBtoKmumu = BrBtoKmumu/1.11_dp
35 |
36 | ! total BR
37 | BrBtoKmumu = BrBtoKmumu*1.0E-7_dp

```

C.7: $\bar{B} \rightarrow X_{d,s} \nu \bar{\nu}$

The branching ratio for $\bar{B} \rightarrow X_q \nu \bar{\nu}$, with $q = d, s$, is given by [105]

$$\text{BR}(\bar{B} \rightarrow X_q \nu \bar{\nu}) = \frac{\alpha^2}{4\pi^2 \sin^4 \theta_W} \frac{|V_{tb} V_{tq}^*|^2}{|V_{cb}|^2} \frac{\text{BR}(\bar{B} \rightarrow X_c e \bar{\nu}_e) \kappa(0)}{f(\hat{m}_c) \kappa(\hat{m}_c)} \times \sum_f \left[(|c_L|^2 + |c_R|^2) f(\hat{m}_q) - 4 \text{Re}(c_L c_R^*) \hat{m}_q \tilde{f}(\hat{m}_q) \right]. \quad (\text{C.68})$$

The sum runs over the three neutrinos and $\hat{m}_i \equiv m_i/m_b$. The functions $f(\hat{m}_c)$ and $\kappa(\hat{m}_c)$ represent the phase-space and the 1-loop QCD corrections, respectively. In case of $\kappa(\hat{m}_c)$, one needs the numerical values $\kappa(0) = 0.83$ and $\kappa(\hat{m}_c) = 0.88$. The functions $f(x)$ and $\tilde{f}(x)$ take the form

$$f(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log x \quad (\text{C.69})$$

$$\tilde{f}(x) = 1 + 9x^2 - 9x^4 - x^6 + 12x^2(1+x^2) \log x. \quad (\text{C.70})$$

Finally, $\text{BR}(\bar{B} \rightarrow X_c e \bar{\nu}_e)_{\text{exp}} = 0.101$ [128] and the coefficients c_L and c_R are given by

$$c_L = n_{BX\nu\nu}^q F_{LL}^V \quad (\text{C.71})$$

$$c_R = n_{BX\nu\nu}^q F_{RL}^V, \quad (\text{C.72})$$

where $(n_{BX\nu\nu}^q)^{-1} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} V_{tb}^* V_{tq}$ is the relative factor between our Wilson coefficients and the ones in [105].

Listing 38 BtoQnunu.m

```

1 NameProcess = "BtoQnunu";
2 NameObservables = {{BrBtoSnunu, 7000, "BR(B->s nu nu)"},
3                    {ratioBtoSnunu, 7001, "BR(B->s nu nu)/BR(B->s nu nu)_SM"},
4                    {BrBtoDnunu, 7002, "BR(B->D nu nu)"},
5                    {ratioBtoDnunu, 7003, "BR(B->D nu nu)/BR(B->D nu nu)_SM"}}};
6
7 NeededOperators = {OddvvVRR, OddvvVLL, OddvvVRL, OddvvVLR,
8                    OddvvVRRSM, OddvvVLLSM, OddvvVRLSM, OddvvVLRSM};
9
10 Body = "BtoQnunu.f90";

```

Listing 39 BtoQnunu.f90

```

1 Complex(dp) :: cL, cR, br, br_SM, cL_SM, cR_SM, norm
2 Real(dp) :: f_mq, tf_mq, kappa_0, kappa_c, f_mc, BrBXeNu, sw2, mq
3 Real(dp) :: prefactor, factor1, factor2, GF
4 Integer :: out, i1, i2
5
6 ! -----
7 ! \bar{B} -> X_{d,s} nu nu
8 ! Observable implemented by W. Porod, F. Staub and A. Vicente
9 ! Based on C. Bobeth et al, NPB 630 (2002) 87 [hep-ph/0112305]
10 ! -----
11
12 kappa_0 = 0.830_dp
13 kappa_c = 0.88_dp
14 f_mc = 0.53_dp
15 BrBXeNu = 0.101_dp ! PDG central value
16
17 sw2 = sinw2_160
18 GF = (Alpha_160*4._dp*Pi/sinW2_160)/mw**2*sqrt2/8._dp
19
20 Do out = 1,2
21   If (out.eq.1) Then ! B -> X_d nu nu
22     mq = mf_d(1)/mf_d(3)
23     norm = Alpha_160*4._dp*GF/sqrt2/(2._dp*pi*sinw2_160)* &
24           & Conjg(CKM_160(3,3)*Conjg(CKM_160(3,1)))
25   Else ! B -> X_s nu nu
26     mq = mf_d(2)/mf_d(3)
27     norm = Alpha_160*4._dp*GF/sqrt2/(2._dp*pi*sinw2_160)* &
28           & Conjg(CKM_160(3,3)*Conjg(CKM_160(3,2)))
29   End if
30
31 ! f and tilde f functions
32 f_mq = 1._dp - 8._dp*mq**2 + 8._dp*mq**6 - &
33       & mq**8 - 24._dp*mq**4*Log(mq)
34 tf_mq = 1._dp + 9._dp*mq**2 - 9._dp*mq**4 - mq**6 + &
35       & 12._dp*mq**2*(1._dp + mq**2)*Log(mq)
36
37 prefactor = Alpha_mz**2/(4._dp*pi**2*sw2**2)*Abs(CKM_160(3,3)/ &
38           & CKM_160(2,3))**2*BrBXeNu/(f_mc*kappa_c)*kappa_0
39 factor1 = f_mq
40 factor2 = - 4._dp*mq*tf_mq
41
42 br = 0._dp
43 br_SM = 0._dp
44
45 Do i1 = 1,3
46   Do i2 = 1,3
47
48     ! BSM
49     cL = OddvvVLL(3,out,i1,i2)/norm
50     cR = OddvvVRL(3,out,i1,i2)/norm
51     br = br + factor1*(Abs(cL)**2 + Abs(cR)**2) + &
52         & factor2*Real(cL*Conjg(cR),dp)
53

```

```

54 ! SM
55 cL = OddvvVLLSM(3,out,i1,i2)/norm
56 cR = OddvvVRLSM(3,out,i1,i2)/norm
57 br_SM = br_SM + factor1*(Abs(cL)**2 + Abs(cR)**2) + &
58         & factor2*Real(cL*Conjg(cR),dp)
59
60 End Do
61 End do
62 If (out.eq.1) Then ! B -> X_d nu nu
63   BrBtoDnunu = prefactor*br*Abs(CKM_160(3,1))**2
64   ratioBtoDnunu = br/br_SM
65 Else ! B -> X_s nu nu
66   BrBtoSnunu = prefactor*br*Abs(CKM_160(3,2))**2
67   ratioBtoSnunu = br/br_SM
68 End if
69 End Do

```

C.8: $K \rightarrow \pi \nu \bar{\nu}$

Following [105], the branching ratios for rare Kaon decays involving neutrinos in the final state can be written as

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 2r_1 r_2 r_{K^+} \sum_f \left[(\text{Im} \lambda_t X_f)^2 + (\text{Re} \lambda_c X_{NL} + \text{Re} \lambda_t X_f)^2 \right] \quad (\text{C.73})$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2r_1 r_{K_L} \sum_f (\text{Im} \lambda_t X_f)^2, \quad (\text{C.74})$$

where the sums are over the three neutrino species, $X_{NL} = 9.78 \cdot 10^{-4}$ is the SM NLO charm correction [48, 129], $\lambda_t = V_{ts}^* V_{td}$ and $\lambda_c = V_{cs}^* V_{cd}$, the coefficients r_1 , r_2 , r_{K^+} and r_{K_L} take the numerical values

$$\begin{aligned}
r_1 &= 1.17 \cdot 10^{-4} \\
r_2 &= 0.24 \\
r_{K^+} &= 0.901 \\
r_{K_L} &= 0.944
\end{aligned} \quad (\text{C.75})$$

and X_f contains the Wilson coefficients contributing to the processes, F_{LL}^V and F_{RL}^V , as

$$X_f = n_{K\pi\nu\nu} \left(F_{LL}^V + F_{RL}^V \right). \quad (\text{C.76})$$

Here $n_{K\pi\nu\nu}^{-1} = \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} V_{ts}^* V_{td}$.

Listing 40 KtoPIInunu.m

```

1 NameProcess = "KtoPIInunu";
2 NameObservables = {{BrKptoPipnunu, 8000, "BR(K^+ -> pi^+ nu nu)"},
3   {ratioKptoPipnunu, 8001, "BR(K^+ -> pi^+ nu nu)/BR(K^+ -> pi^+ nu \leftrightarrow
4     \leftarrow nu)_SM"},
5   {BrKltoPinunu, 8002, "BR(K_L -> pi^0 nu nu)"},
6   {ratioKltoPinunu, 8003, "BR(K_L -> pi^0 nu nu)/BR(K_L -> pi^0 nu \leftrightarrow
7     \leftarrow nu)_SM"}};
8
9 NeededOperators = {OddvvVRR, OddvvVLL, OddvvVRL, OddvvVLR,
10   OddvvVRRSM, OddvvVLLSM, OddvvVRLSM, OddvvVLRSM};
11
12 Body = "KtoPIInunu.f90";

```

Listing 41 KtoPInunu.f90

```

1 Complex(dp) :: br, r1, r2, rKp, rKl, Xx, XNL, Lt, Lc
2 Complex(dp) :: Xx_SM, br_SM, norm
3 Real(dp) :: GF
4 Integer :: out, i1, i2
5
6 ! -----
7 ! K -> pi nu nu
8 ! Observable implemented by W. Porod, F. Staub and A. Vicente
9 ! Based on C. Bobeth et al, NPB 630 (2002) 87 [hep-ph/0112305]
10 ! -----
11
12 GF = (Alpha_160*4._dp*Pi/sinW2_160)/mw**2*sqrt2/8._dp
13 norm = Alpha_160*4._dp*GF/sqrt2/(2._dp*pi*sinw2_160) &
14       & *Conjg(CKM_160(3,2))*CKM_160(3,1)
15
16 r1 = 1.17E-4_dp
17 r2 = 0.24_dp
18 rKp = 0.901
19 rKl = 0.944
20
21 ! SM NLO charm correction
22 ! See G. Buchalla and A. Buras, NPB 412 (1994) 106 and NPB 548 (1999) 309
23 XNL = 9.78E-4_dp
24
25 ! out = 1 : K^+ -> pi^+ nu nu
26 ! out = 2 : K_L -> pi^0 nu nu
27
28 Do out = 1,2
29   br = 0._dp
30   br_SM = 0._dp
31   Do i1 = 1,3
32     Do i2 = 1,3
33       Xx = ((OddvvVLL(2,1,i1,i2)+OddvvVRL(2,1,i1,i2))/norm)
34       Xx_SM = ((OddvvVLLSM(2,1,i1,i2)+OddvvVRLSM(2,1,i1,i2))/norm)
35       Lt = Conjg(CKM_160(3,2))*CKM_160(3,1)
36       Lc = Conjg(CKM_160(2,2))*CKM_160(2,1)
37       If (out.eq.1) Then
38         br = br + Aimag(Xx*Lt)**2 + (Real(Lc*XNL,dp) + Real(Xx*Lt,dp))**2
39         br_SM = br_SM + Aimag(Xx_SM*Lt)**2 + &
40             & (Real(Lc*XNL,dp) + Real(Xx_SM*Lt,dp))**2
41       Else
42         br = br + Abs(Aimag(Xx*Lt))**2
43         br_SM = br_SM + Abs(Aimag(Xx_SM*Lt))**2
44       End if
45     End Do
46   End Do
47   If (out.eq.1) Then ! K^+ -> pi^+ nu nu
48     BrKptoPipnunu = 2._dp*r1*r2*rKp*br
49     RatioKptoPipnunu = br/br_SM
50     ! SM expectation: (7.2 +/- 2.1)*10^-11 (hep-ph/0112135)
51   Else ! K_L -> pi^0 nu nu
52     BrKltoPinunu = 2._dp*r1*rKl*br
53     RatioKltoPinunu = br/br_SM
54     ! SM expectation: (3.1 +/- 1.0)*10^-11 (hep-ph/0408142)
55   End if
56 End Do

```

C.9: $\Delta M_{B_{s,d}}$

The $B_q^0 - \bar{B}_q^0$ mass difference can be written as [108, 130]

$$\Delta M_{B_q} = \frac{G_F^2 m_W^2}{6\pi^2} m_{B_q} \eta_B f_{B_q}^2 \hat{B}_{B_q} |V_{tq}^{\text{eff}}|^2 |F_{tt}^q|, \quad (\text{C.77})$$

where $q = s, d$, m_{B_q} and f_{B_q} are the B_q^0 mass and decay constant, respectively, $\eta_B = 0.55$ is a QCD factor [47, 131], \hat{B}_{B_q} is a non-perturbative parameter (with values $\hat{B}_{B_d} = 1.26$ and $\hat{B}_{B_s} = 1.33$, obtained from recent lattice computations [132]) and $|V_{tq}^{\text{eff}}|^2 = (V_{tb}^* V_{tq})^2$. F_{tt}^q is given by

$$\begin{aligned} F_{tt}^q &= S_0(x_t) + \frac{1}{4r} C_{\text{new}}^{VLL} \\ &+ \frac{1}{4r} C_1^{VRR} + \bar{P}_1^{LR} C_1^{LR} + \bar{P}_2^{LR} C_2^{LR} \\ &+ \bar{P}_1^{SLL} (C_1^{SLL} + C_1^{SRR}) + \bar{P}_2^{SLL} (C_2^{SLL} + C_2^{SRR}) \end{aligned} \quad (\text{C.78})$$

where $r = 0.985$ [47], $x_t = \frac{m_t^2}{m_W^2}$, with m_t the top quark mass, the \bar{P} coefficients take the numerical values

$$\begin{aligned} \bar{P}_1^{LR} &= -0.71 \\ \bar{P}_2^{LR} &= 0.90 \\ \bar{P}_1^{SLL} &= -0.37 \\ \bar{P}_2^{SLL} &= -0.72 \end{aligned} \quad (\text{C.79})$$

and the function

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1 - x_t)^2} - \frac{3x_t^3 \log x_t}{2(1 - x_t)^3} \quad (\text{C.80})$$

was introduced by Inami and Lim in [133] and given, for example, in [134]. Finally, the coefficients in Eq. (C.78) are related to the D_{XY}^I coefficients in Eq.(A.13) as

$$C_{\text{new}}^{VLL} = n_{\Delta}^q (D_{LL}^V - D_{LL}^{V,\text{SM}}) \quad (\text{C.81})$$

$$C_1^{VRR} = n_{\Delta}^q D_{RR}^V \quad (\text{C.82})$$

$$C_1^{LR} = n_{\Delta}^q (D_{LR}^V + D_{RL}^V) \quad (\text{C.83})$$

$$C_2^{LR} = n_{\Delta}^q (D_{LR}^S + D_{RL}^S + \delta_2^{LR}) \quad (\text{C.84})$$

$$C_1^{SLL} = n_{\Delta}^q (D_{LL}^S + \delta_1^{SLL}) \quad (\text{C.85})$$

$$C_1^{SRR} = n_{\Delta}^q (D_{RR}^S + \delta_1^{SRR}) \quad (\text{C.86})$$

$$C_2^{SLL} = n_{\Delta}^q D_{LL}^T \quad (\text{C.87})$$

$$C_2^{SRR} = n_{\Delta}^q D_{RR}^T \quad (\text{C.88})$$

where the factor $(n_{\Delta}^q)^{-1} = \frac{G_F^2 m_W^2}{16\pi^2} |V_{tq}^{\text{eff}}|^2$ normalizes our Wilson coefficients to the ones in [108, 130]. The corrections δ_2^{LR} , δ_1^{SLL} and δ_1^{SRR} are induced by double penguin diagrams mediated by scalar and pseudoscalar states [108, 130]. These 2-loop contributions may have a sizable impact in some models, and their inclusion is necessary in order to achieve a precise result for ΔM_{B_q} . They can be written as

$$\delta_2^{LR} = - \frac{H_L^{S,P} (H_R^{S,P})^*}{m_{S,P}^2} \quad (\text{C.89})$$

$$\delta_1^{SLL} = - \frac{(H_L^{S,P})^2}{2m_{S,P}^2} \quad (\text{C.90})$$

$$\delta_1^{SRR} = - \frac{(H_L^{S,P})^2}{2m_{S,P}^2} \quad (\text{C.91})$$

where $H_L^{S,P}$ and $H_R^{S,P}$ are defined in Eq.(A.17). The double penguin corrections in Eqs.(C.89)-(C.91) are obtained by summing up over all scalar and pseudoscalar states in the model.

Listing 42 DeltaMBq.m

```

1 NameProcess = "DeltaMBq";
2 NameObservables = {{DeltaMBs, 1900, "Delta(M_Bs)"},
3                   {ratioDeltaMBs, 1901, "Delta(M_Bs)/Delta(M_Bs)_SM"},
4                   {DeltaMBq, 1902, "Delta(M_Bd)"},
5                   {ratioDeltaMBq, 1903, "Delta(M_Bd)/Delta(M_Bd)_SM"}}};
6
7 ExternalStates = {Fd};
8 NeededOperators = {O4dSLL, O4dSRR, O4dSRL, O4dSLR, O4dVRR, O4dVLL,
9                   O4dVLLSM, O4dVRL, O4dVLR, O4dTLL, O4dTLR, O4dTRL, O4dTRR};
10
11 IncludeSMPrediction["DeltaMBq"] = False;
12
13 Body = "DeltaMBq.f90";

```

Listing 43 DeltaMBq.f90

```

1 Complex(dp) :: MBq, etaB, FBq2, BBq, Ftt, Veff2, r, &
2   & P1bLR, P2bLR, P1bSLL, P2bSLL, norm, &
3   & CVLLnew, CIVRR, C1LR, C2LR, C1SLL, C1SRR, C2SLL, C2SRR
4 Real(dp) :: hbar, xt, GF
5 Real(dp) :: mS
6 Complex(dp) :: HL, HR, AL, AR
7 Integer :: i1, iS
8
9 ! -----
10 ! Delta M_{Bd,Bs}
11 ! Observable implemented by W. Porod, F. Staub and A. Vicente
12 ! Based on A. J. Buras et al, NPB 619 (2001) 434 [hep-ph/0107048]
13 ! and NPB 659 (2003) 3 [hep-ph/0210145]
14 ! -----
15
16 hbar = 6.58211889e-25_dp
17 xt = mf_u2_160(3)/mw2
18 r = 0.985_dp
19 P1bLR = -0.71_dp
20 P2bLR = 0.90_dp
21 P1bSLL = -0.37_dp
22 P2bSLL = -0.72_dp
23
24 ! QCD factor, see A. J. Buras et al, NPB 47 (1990) 491
25 ! and J. Urban et al, NPB 523 (1998) 40
26 etaB = 0.55_dp
27
28 GF = (Alpha_160*4._dp*Pi/sinW2_160)/mw**2*sqrt2/8._dp
29
30 Do i1 = 1,2
31
32   If (i1.eq.1) Then ! Delta M_Bd
33     MBq = mass_B0d
34     FBq2 = f_B0d_CONST**2
35     BBq = 1.26_dp ! see arXiv:0910.2928
36     Veff2 = Conjg(Conjg(CKM_160(3,3))*CKM_160(3,1))*2
37   Else ! Delta M_Bs
38     MBq = mass_B0s
39     FBq2 = f_B0s_CONST**2
40     BBq = 1.33_dp ! see arXiv:0910.2928
41     Veff2 = Conjg(Conjg(CKM_160(3,3))*CKM_160(3,2))*2
42   End if
43
44   ! normalization factor
45   norm = GF**2*mw2/(16._dp*Pi**2)*Veff2
46
47   ! Wilson coefficients
48   CVLLnew = (O4dVLL(3,i1,3,i1)-O4dVLLSM(3,i1,3,i1))/norm ! we remove the SM contribution
49   CIVRR = O4dVRR(3,i1,3,i1)/norm
50   C1LR = (O4dVLR(3,i1,3,i1)+O4dVRL(3,i1,3,i1))/norm

```

```

51 C2LR = (O4dSLR(3,i1,3,i1)+O4dSRL(3,i1,3,i1))/norm
52 C1SLL = O4dSLL(3,i1,3,i1)/norm
53 C1SRR = O4dSRR(3,i1,3,i1)/norm
54 C2SLL = O4dTLL(3,i1,3,i1)/norm
55 C2SRR = O4dTRR(3,i1,3,i1)/norm
56
57
58 ! Double Higgs penguins
59 @ If [getGen[HiggsBoson] > 1, "Do iS = 1, "<>ToString[getGen[HiggsBoson]],"]
60 @ If [getGen[HiggsBoson] > 1, "HL = OH2qSL(3,i1,iS)", "HL = OH2qSL(3,i1)"]
61 @ If [getGen[HiggsBoson] > 1, "HR = OH2qSR(3,i1,iS)", "HR = OH2qSR(3,i1)"]
62 @ If [getGen[HiggsBoson] > 1, "mS = "<>SPhenoMassSq[HiggsBoson,iS], "mS = <=>
    <=> "<>SPhenoMassSq[HiggsBoson]]
63 C2LR = C2LR - HL*Conjg(HR)/(mS*norm)
64 C1SLL = C1SLL - 0.5_dp*HL**2/(mS*norm)
65 C1SRR = C1SRR - 0.5_dp*HR**2/(mS*norm)
66 @ If [getGen[HiggsBoson] > 1,"End Do","]"]
67
68
69 @ If [getGen[PseudoScalar] > 1, "Do iS = <=>
    <=> "<>ToString[getGenSPhenoStart[PseudoScalar]]<>", <=>
    <=> "<>ToString[getGen[PseudoScalar]],"]"]
70 @ If [getGen[PseudoScalar] > 1, "AL = OAh2qSL(3,i1,iS)", "AL = OAh2qSL(3,i1)"]
71 @ If [getGen[PseudoScalar] > 1, "AR = OAh2qSR(3,i1,iS)", "AR = OAh2qSR(3,i1)"]
72 @ If [getGen[PseudoScalar] > 1, "mS = "<>SPhenoMassSq[PseudoScalar,iS], "mS = <=>
    <=> "<>SPhenoMassSq[PseudoScalar]]
73 C2LR = C2LR - AL*Conjg(AR)/(mS*norm)
74 C1SLL = C1SLL - 0.5_dp*AL**2/(mS*norm)
75 C1SRR = C1SRR - 0.5_dp*AR**2/(mS*norm)
76 @ If [getGen[PseudoScalar] > 1,"End Do","]"]
77
78
79 Ftt = S0xt(xt) + CVLLnew/(4._dp*r) + &
80 & C1VRR/(4._dp*r) + P1bLR*C1LR + P2bLR*C2LR + &
81 & P1bSLL*(C1SLL + C1SRR) + P2bSLL*(C2SLL + C2SRR)
82
83 If (i1.eq.1) Then ! Delta M_Bd
84   ratioDeltaMBq = Abs(Ftt/S0xt(xt))
85   DeltaMBq = G_F**2*mw2/(6._dp*Pi**2)* &
86   & MBq*etaB*BBq*FBq2*Veff2*Abs(Ftt)*1.e-12_dp/hbar
87 Else ! Delta M_Bs
88   ratioDeltaMBs = Abs(Ftt/S0xt(xt))
89   DeltaMBs = G_F**2*mw2/(6._dp*Pi**2)* &
90   & MBq*etaB*BBq*FBq2*Veff2*Abs(Ftt)*1.e-12_dp/hbar
91 End if
92
93 End Do
94
95 Contains
96
97 Real(dp) Function S0xt(x) ! See for example hep-ph/9806471
98   Implicit None
99   Real(dp), Intent(in) :: x
100   S0xt = 1._dp - 2.75_dp * x + 0.25_dp * x**2 &
101   & - 1.5_dp * x**2 * Log(x) / (1-x)
102   S0xt = x*S0xt / (1-x)**2
103 End Function S0xt

```

C.10: ΔM_K and ε_K

ΔM_K and ε_K , the observables associated to $K^0 - \bar{K}^0$ mixing, can be written as [9, 134]

$$\Delta M_K = 2 \operatorname{Re} \langle \bar{K}^0 | H_{\text{eff}}^{\Delta S=2} | K^0 \rangle \quad (\text{C.92})$$

$$\varepsilon_K = \frac{e^{i\pi/4}}{\sqrt{2}\Delta M_K} \operatorname{Im} \langle \bar{K}^0 | H_{\text{eff}}^{\Delta S=2} | K^0 \rangle. \quad (\text{C.93})$$

The matrix element in Eqs. (C.92) and (C.93) is given by

$$\begin{aligned} \langle \bar{K}^0 | H_{\text{eff}}^{\Delta S=2} | K^0 \rangle = & f_V \left(D_{LL}^V + D_{RR}^V \right) + f_S \left(D_{LL}^S + D_{RR}^S \right) + f_T \left(D_{LL}^T + D_{RR}^T \right) \\ & + f_{LR}^1 \left(D_{LR}^S + D_{RL}^S \right) + f_{LR}^2 \left(D_{LR}^V + D_{RL}^V \right). \end{aligned} \quad (\text{C.94})$$

The f coefficients are

$$f_V = \frac{1}{3} m_K f_K^2 B_1^{VLL}(\mu) \quad (\text{C.95})$$

$$f_S = -\frac{5}{24} \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2 m_K f_K^2 B_1^{SLL}(\mu) \quad (\text{C.96})$$

$$f_T = -\frac{1}{2} \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2 m_K f_K^2 B_2^{SLL}(\mu) \quad (\text{C.97})$$

$$f_{LR}^1 = -\frac{1}{6} \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2 m_K f_K^2 B_1^{LR}(\mu) \quad (\text{C.98})$$

$$f_{LR}^2 = \frac{1}{4} \left(\frac{m_K}{m_s(\mu) + m_d(\mu)} \right)^2 m_K f_K^2 B_2^{LR}(\mu) \quad (\text{C.99})$$

where $\mu = 2$ GeV is the energy scale at which the matrix element is computed and f_K the Kaon decay constant. The values of the quark masses at $\mu = 2$ GeV are given by $m_d(\mu) = 7$ MeV and $m_s(\mu) = 125$ MeV (see table 1 in [98]), whereas the B_i^X coefficients have the following values at $\mu = 2$ GeV [135]: $B_1^{VLL}(\mu) = 0.61$, $B_1^{SLL}(\mu) = 0.76$, $B_2^{SLL}(\mu) = 0.51$, $B_1^{LR}(\mu) = 0.96$ and $B_2^{LR}(\mu) = 1.3$.

As in [9], we treat the SM contribution separately. We define $D_{LL}^V = D_{LL}^{V,SM} + D_{LL}^{V,BSM}$. For $D_{LL}^{V,BSM}$ one just subtracts the SM contributions to D_{LL}^V , whereas for $D_{LL}^{V,SM}$ one can use the results in [136–138], where the relevant QCD corrections are included,

$$D_{LL}^{V,SM} = \frac{G_F^2 m_W^2}{4\pi^2} \left[\lambda_c^{*2} \eta_1 S_0(x_c) + \lambda_t^{*2} \eta_2 S_0(x_t) + 2\lambda_c^* \lambda_t^* \eta_3 S_0(x_c, x_t) \right]. \quad (\text{C.100})$$

Here $x_i = m_i^2/m_w^2$, $\lambda_i = V_{is}^* V_{id}$ and $S_0(x)$ and $S_0(x, y)$ are the Inami-Lim functions [133]. $S_0(x)$ was already defined in Eq. (C.80), whereas $S_0(x_c, x_t)$ is given by [134]

$$S_0(x_c, x_t) = x_c \left[\log \frac{x_t}{x_c} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \log x_t}{4(1-x_t)^2} \right]. \quad (\text{C.101})$$

In the last expression we have kept only terms linear in $x_c \ll 1$. Finally, the η_i coefficients comprise short distance QCD corrections. Their numerical values are $\eta_{1,2,3} = (1.44, 0.57, 0.47)$ [138]¹³.

Listing 44 KKmix.m

```

1 NameProcess = "KKmix";
2 NameObservables = {{DeltaMK, 9100, "Delta(M_K)"},
3                     {ratioDeltaMK, 9102, "Delta(M_K)/Delta(M_K)_SM"},
4                     {epsK, 9103, "epsilon_K"},
5                     {ratioepsK, 9104, "epsilon_K/epsilon_K^SM"}};
6
7 NeededOperators = {O4dSLL, O4dSRR, O4dSRL, O4dSLR, O4dVRR, O4dVLL, O4dVRL,
8                     O4dVLR, O4dTLL, O4dTLR, O4dTRL, O4dTRR,
9                     O4dSLLSM, O4dSRRSM, O4dSRLSM, O4dSLRSM, O4dVRRSM, O4dVLLSM, ↔
10                     ↔ O4dVRLSM, O4dVLRSM,
11                     O4dTLLSM, O4dTLRSM, O4dTRLSM, O4dTRRSM};
12 Body = "KKmix.f90 ";

```

¹³Note that we have chosen a value for η_1 which results from our numerical values for $\alpha_s(m_Z)$ and $m_c(m_c)$, see table 5 in [138].

Listing 45 KKmix.f90

```

1 Real(dp) :: b_VLL, b_SLL1, b_SLL2, b_LR1, b_LR2
2 Real(dp) :: ms_mu, md_mu
3 Complex(dp) :: CVLL, CVRR, CSLL, CSRR, CTLL, CTRR, CLR1, CLR2
4 Complex(dp) :: fV, fS, fT, fLR1, fLR2, cVLLSM
5 Complex(dp) :: f_k, M_K, H2eff, DeltaMK_SM, epsK_SM
6 Real(dp) :: norm, hbar, xt, xc, GF
7 Integer :: il
8 Real(dp), Parameter :: eta_tt = 0.57_dp, eta_ct = 0.47_dp, &
9 & eta_cc = 1.44_dp
10 ! Parameters from S. Herrlich and U. Nierste NPB 476 (1996) 27
11
12 ! -----
13 ! Delta M K and epsilon_K
14 ! Observables implemented by W. Porod, F. Staub and A. Vicente
15 ! Based on A. Crivellin et al, Comput. Phys. Commun. 184 (2013) 1004 [arXiv:1203.5023]
16 ! -----
17
18 ! using globally defined hadronic parameters
19 M_K = mass_K0
20 f_k = f_k_CONST
21
22 xt = mf_u(3)**2 / mW**2
23 xc = mf_u(2)**2 / mW**2
24
25 GF = (Alpha_160*4._dp*Pi/sinW2_160)/mw**2*sqrt2/8._dp
26
27 ! Coefficients at mu = 2 GeV
28 ! See A. J. Buras et al, NPB 605 (2001) 600 [hep-ph/0102316]
29 b_VLL = 0.61_dp
30 b_SLL1 = 0.76_dp
31 b_SLL2 = 0.51_dp
32 b_LR1 = 0.96_dp
33 b_LR2 = 1.3_dp
34
35 ! Quark mass values at mu = 2 GeV
36 ! See M. Ciuchini et al, JHEP 9810 (1998) 008 [hep-ph/9808328] - Table 1
37 md_mu = 0.007_dp
38 ms_mu = 0.125_dp
39
40 fV = 1._dp/3._dp*M_K*f_k**2*b_VLL
41 fS = -5._dp/24._dp*M_K*f_k**2*(M_K/(ms_mu+md_mu))**2*b_SLL1
42 fT = -1._dp/2._dp*M_K*f_k**2*(M_K/(ms_mu+md_mu))**2*b_SLL2
43 fLR1 = -1._dp/6._dp*M_K*f_k**2*(M_K/(ms_mu+md_mu))**2*b_LR1
44 fLR2 = 1._dp/4._dp*M_K*f_k**2*(M_K/(ms_mu+md_mu))**2*b_LR2
45
46 ! SM contribution
47 ! Based on the results by S. Herrlich and U. Nierste
48 ! NPB 419 (1994) 292, PRD 52 (1995) 6505 and NPB 476 (1996) 27
49 cVLLSM = eta_cc * (Conjg(CKM_160(2,2))*CKM_160(2,1))**2 * S0xt(xc) &
50 & + eta_tt * (Conjg(CKM_160(3,2))*CKM_160(3,1))**2 * S0xt(xt) &
51 & + Conjg(CKM_160(2,2))*CKM_160(3,2))*(CKM_160(2,1)*CKM_160(3,1)) &
52 & * 2._dp * eta_ct * S0_2(xc,xt)
53
54 cVLLSM = Conjg(cVLLSM) ! we compute (d\bar{s})(d\bar{s}) and not (\bar{d}s)(\bar{d}s)
55 cVLLSM = oo4pi2*(GF*mW)**2*cVLLSM ! normalization
56
57 ! BSM contributions (+SM in CVLL)
58 CVLL = O4dVLL(2,1,2,1)-O4dVLLSM(2,1,2,1)+cVLLSM
59 CVRR = O4dVRR(2,1,2,1)
60 CSLL = O4dSLL(2,1,2,1)
61 CSRR = O4dSRR(2,1,2,1)
62 CTLL = O4dTLL(2,1,2,1)
63 CTRR = O4dTRR(2,1,2,1)
64 CLR1 = O4dSLR(2,1,2,1)+O4dSRL(2,1,2,1)
65 CLR2 = O4dVLR(2,1,2,1)+O4dVRL(2,1,2,1)
66
67 ! BSM

```

```

68 H2eff = fV*(CVLL+CVRR) + fS*(CSLL+CSRR) +fT*(CTLL+CTRR) &
69      & + fLR1*CLR1 + fLR2*CLR2
70
71 DeltaMK = Abs(2._dp*Real(H2eff,dp))
72 epsK = 1._dp/(sqrt2*DeltaMK)*Abs(Aimag(H2eff))
73
74 ! SM
75 H2eff = fV*cVLLSM
76
77 DeltaMK_SM = Abs(2._dp*Real(H2eff,dp))
78 epsK_SM = 1._dp/(sqrt2*DeltaMK_SM)*Abs(Aimag(H2eff))
79
80 ratioDeltaMK = DeltaMK/DeltaMK_SM
81 ratioepsK = epsK/epsK_SM
82
83 Contains
84
85 ! Inami - Lim functions
86
87 Real(dp) Function S0xt(x)
88   Implicit None
89   Real(dp), Intent(in) :: x
90   S0xt = 1._dp - 2.75_dp * x + 0.25_dp * x**2 - &
91      & 1.5_dp * x**2 * Log(x) / (1-x)
92   S0xt = x*S0xt / (1-x)**2
93 End Function S0xt
94
95 Real(dp) Function S0_2(xc, xt)
96   Implicit None
97   Real(dp), Intent(in) :: xc, xt
98   S0_2 = Log(xt/xc) - 0.75_dp * xt / (1-xt) &
99      & - 0.75_dp * xt**2 * Log(xt) / (1-xt)**2
100   S0_2 = xc * S0_2
101 End Function S0_2

```

C.11: $P \rightarrow \ell \nu$

Although $P \rightarrow \ell \nu$, where $P = qq'$ is a pseudoscalar meson, does not violate quark flavor, we have included it in the list of observables for practical reasons, as it can be computed with the same ingredients as the QFV observables. The decay width for the process $P \rightarrow \ell_\alpha \nu$ is given by [139]

$$\begin{aligned}
 \Gamma(P \rightarrow \ell_\alpha \nu) = & \frac{|G_F f_P (m_P^2 - m_{\ell_\alpha}^2)|^2}{8\pi m_P^3} \\
 & \times \sum_\nu \left| V_{qq'} m_{\ell_\alpha} + \frac{m_{\ell_\alpha}}{2\sqrt{2}} (G_{LL}^V - G_{RL}^V) + \frac{m_P^2}{2\sqrt{2}(m_q + m_{q'})} (G_{RR}^S - G_{LR}^S) \right|^2.
 \end{aligned} \tag{C.102}$$

Here f_P is the meson decay constant, m_q and $m_{q'}$ are the masses of the quarks in the meson and the Wilson coefficients G_{XY}^I are defined in Eq.(A.16). The sum in Eq.(C.102) is over the three neutrinos (whose masses are neglected).

Each $P \rightarrow \ell_\alpha \nu$ decay width is plagued by hadronic uncertainties. However, by taking the ratios

$$R_P = \frac{\Gamma(P \rightarrow e \nu)}{\Gamma(P \rightarrow \mu \nu)} \tag{C.103}$$

the hadronic uncertainties cancel out to a good approximation, allowing for a precise theoretical determination. In case of R_K , the SM prediction includes small electromagnetic corrections that account for internal bremsstrahlung and structure-dependent effects [140]. This leads to an impressive theoretical uncertainty of $\delta R_K / R_K \sim 0.1\%$, making R_P the perfect observable to search for lepton flavor universality violation [141].

Listing 46 Plnu.m

```

1 NameProcess = "Plnu";
2 NameObservables = {{BrDmunu, 300, "BR(D->mu nu)"},
3                     {ratioDmunu, 301, "BR(D->mu nu)/BR(D->mu nu)_SM"},
4                     {BrDsmunu, 400, "BR(Ds->mu nu)"},
5                     {ratioDsmunu, 401, "BR(Ds->mu nu)/BR(Ds->mu nu)_SM"},
6                     {BrDstauunu, 402, "BR(Ds->tau nu)"},
7                     {ratioDstauunu, 403, "BR(Ds->tau nu)/BR(Ds->tau nu)_SM"},
8                     {BrBmunu, 500, "BR(B->mu nu)"},
9                     {ratioBmunu, 501, "BR(B->mu nu)/BR(B->mu nu)_SM"},
10                    {BrBtaunu, 502, "BR(B->tau nu)"},
11                    {ratioBtaunu, 503, "BR(B->tau nu)/BR(B->tau nu)_SM"},
12                    {BrKmunu, 600, "BR(K->mu nu)"},
13                    {ratioKmunu, 601, "BR(K->mu nu)/BR(K->mu nu)_SM"},
14                    {RK, 602, "R_K = BR(K->e nu)/(K->mu nu)"},
15                    {RKSM, 603, "R_K^SM = BR(K->e nu)_SM/(K->mu nu)_SM"}}};
16
17 NeededOperators = {OduLvSLL, OduLvSRR, OduLvSRL, OduLvSLR,
18                   OduLvVRR, OduLvVLL, OduLvVRL, OduLvVLR,
19                   OduLvSLLSM, OduLvSRRSM, OduLvSRLSM, OduLvSLRSM,
20                   OduLvVRRSM, OduLvVLLSM, OduLvVRLSM, OduLvVLRSM
21 };
22
23 Body = "Plnu.f90";

```

Listing 47 Plnu.f90

```

1 Integer :: gt1, gt2, i1, i2, iP
2 Complex(dp) :: br, br_SM
3 Real(dp) :: m_M, f_M, tau_M, mlep, mq1, mq2, hbar, ratio, &
4           & BrKenuSM, BRKenu, QED
5
6 ! -----
7 ! P -> l nu
8 ! Observable implemented by W. Porod, F. Staub and A. Vicente
9 ! Based on J. Barranco et al, arXiv:1303.3896
10 ! -----
11
12 hbar = 6.58211889e-25_dp
13
14 ! Electromagnetic correction to R_K
15 ! See V. Cirigliano, I. Rosell, PRL 99 (2007) 231801 [arXiv:0707.3439]
16 QED = -3.6e-2_dp
17
18 ! meson parameters
19
20 Do iP=1,4
21 If (iP.eq.1) Then ! Ds-meson
22   gt1 = 2
23   gt2 = 2
24   m_M = mass_Dsp
25   f_M = f_DSp_CONST
26   tau_M = tau_DSp/hbar
27 ElseIf (iP.eq.2) Then ! B-meson
28   gt1 = 3
29   gt2 = 1
30   m_M = mass_Bp
31   f_M = f_Bp_CONST
32   tau_M = tau_Bp/hbar
33 ElseIf (iP.eq.3) Then ! Kaon
34   gt1 = 2
35   gt2 = 1
36   m_M = mass_Kp
37   f_M = f_Kp_CONST
38   tau_M = tau_Kp/hbar
39 ElseIf (iP.eq.4) Then ! D-meson
40   gt1 = 1
41   gt2 = 2
42   m_M = mass_Dp

```

```

43  f_M = f_Dp_CONST
44  tau_M = tau_Dp/hbar
45  End if
46
47  mq1 = mf_u_160(gt2)
48  mq2 = mf_d_160(gt1)
49
50  Do i1=1,3
51  br = 0._dp
52  br_SM = 0._dp
53  mlep = mf_l(i1)
54
55  Do i2=1,3
56  br = br + ((OdulvVLL(gt1,gt2,i1,i2)-OdulvVLR(gt1,gt2,i1,i2))*mlep/      &
57              & (2._dp*sqrt2))      &
58  & + m_M**2*(OdulvSRL(gt1,gt2,i1,i2)-OdulvSLL(gt1,gt2,i1,i2))/      &
59              & (2._dp*sqrt2*(mq1+mq2)))      &
60  br_SM = br_SM+ (OdulvVLLSM(gt1,gt2,i1,i2)-OdulvVLRSM(gt1,gt2,i1,i2)) &
61              & *mlep/(2._dp*sqrt2)
62  End Do
63
64  ratio = Abs(br/br_SM)**2
65  br = oo8pi*tau_M*(f_M)**2*M_M*Abs(br)**2*(1._dp - mlep**2/M_M**2)**2 ! G_F already ⇔
        ⇔ in coefficients included
66
67
68  If (iP.eq.1) Then !! Ds-meson
69  If (i1.eq.2) Then ! Ds->mu nu
70  BrDsmunu = br
71  ratioDsmunu = ratio
72  Elseif (i1.eq.3) Then ! Ds->tau nu
73  BrDstaunu = br
74  ratioDstaunu = ratio
75  End if
76  Elseif (iP.eq.2) Then !! B-meson
77  If (i1.eq.2) Then ! B->mu nu
78  BrBmunu = br
79  ratioBmunu = ratio
80  Else ! B->tau nu
81  BrBtaunu = br
82  ratioBtaunu = ratio
83  End if
84  Else If (iP.eq.3) Then !! Kaon
85  If (i1.eq.1) Then ! K->e nu
86  BrKenu = br
87  BrKenuSM = BrKenu*ratio
88  Elseif (i1.eq.2) Then ! K->mu nu
89  BrKmunu = br
90  ratioKmunu = ratio
91  RK = BrKenu/BrKmunu*(1+QED)
92  RKSM = BrKenuSM/BrKmunu*ratio*(1+QED)
93  End if
94  Else If (iP.eq.4) Then !! D-meson
95  If (i1.eq.2) Then ! D->mu nu
96  BrDmunu = br
97  ratioDmunu = ratio
98  End if
99  End if
100 End Do
101 End Do

```

D: Models

The following models are included in the public version of **SARAH** and can now be used together with the **FlavorKit** to get predictions for the different observables.

D.1: Supersymmetric Models

- Minimal supersymmetric standard model (see Ref. [142] and references therein)
 - With general flavor and CP structure (**MSSM**)
 - Without flavor violation (**MSSM/NoFV**)
 - With explicit CP violation in the Higgs sector (**MSSM/CPV**)
 - In SCKM basis (**MSSM/CKM**)
- Singlet extensions:
 - Next-to-minimal supersymmetric standard model (**NMSSM**, **NMSSM/NoFV**, **NMSSM/CPV**, **NMSSM/CKM**) (see Refs. [143, 144] and references therein)
 - near-to-minimal supersymmetric standard model (**near-MSSM**) [145]
 - General singlet extended, supersymmetric standard model (**SMSSM**) [145, 146]
 - DiracNMSSM (**DiracNMSSM**) [147, 148]
- Triplet extensions
 - Triplet extended MSSM (**TMSSM**) [149]
 - Triplet extended NMSSM (**TNMSSM**) [150]
- Models with R -parity violation [151–158]
 - bilinear R_pV (**MSSM-RpV/Bi**)
 - Lepton number violation (**MSSM-RpV/LnV**)
 - Only trilinear lepton number violation (**MSSM-RpV/TriLnV**)
 - Baryon number violation (**MSSM-RpV/BnV**)
 - $\mu\nu$ SSM (**munuSSM**) [159, 160]
- Additional $U(1)'$ s
 - $U(1)$ -extended MSSM (**UMSSM**) [145]
 - secluded MSSM (**secluded-MSSM**) [161]
 - minimal $B - L$ model (**B-L-SSM**) [162–165]
 - minimal singlet-extended $B - L$ model (**N-B-L-SSM**)
- SUSY-scale seesaw extensions
 - inverse seesaw (**inverse-Seesaw**) [166, 167]
 - linear seesaw (**LinSeesaw**) [166, 168]
 - singlet extended inverse seesaw (**inverse-Seesaw-NMSSM**) [169]
 - inverse seesaw with $B - L$ gauge group (**B-L-SSM-IS**) [170]
 - minimal $U(1)_R \times U(1)_{B-L}$ model with inverse seesaw (**BLRinvSeesaw**) [74, 171]
- Models with Dirac Gauginos
 - MSSM/NMSSM with Dirac Gauginos (**DiracGauginos**) [172–174]
 - minimal R -Symmetric SSM (**MRSSM**) [175]
 - Minimal Dirac Gaugino supersymmetric standard model (**MDGSSM**) [86]
- High-scale extensions
 - Seesaw 1 - 3 ($SU(5)$ version) , (**Seesaw1**, **Seesaw2**, **Seesaw3**) [63, 65, 68, 176, 177]
 - Left/right model (Ω LR) (**Omega**) [178, 179]
 - Quiver model (**QEW12**, **QEWm1d2L3**) [180]

D.2: Non-Supersymmetric Models

- Standard Model (SM) (**SM**), Standard model in CKM basis (**SM/CKM**) (see for instance Ref. [181] and references therein)
- inert Higgs doublet model (**Inert**) [182]
- B-L extended SM (**B-L-SM**) [183–185]
- B-L extended SM with inverse seesaw (**B-L-SM-IS**) [186]
- SM extended by a scalar color octet (**SM-8C**) [187]
- Two Higgs doublet model (**THDM**) (see for instance Ref. [188] and references therein)
- Singlet extended SM (**SSM**) [189]
- Singlet Scalar DM (**SSDM**) [190]

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